Adapting Guidance Methodologies for Trajectory Generation in Entry Shape Optimization

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Motivation

Flight Feasible Trajectories will

Model **Realistic In-Flight Thermal States:**

- Allow for increased accuracy in Thermal Protection System sizing (potential mass savings)

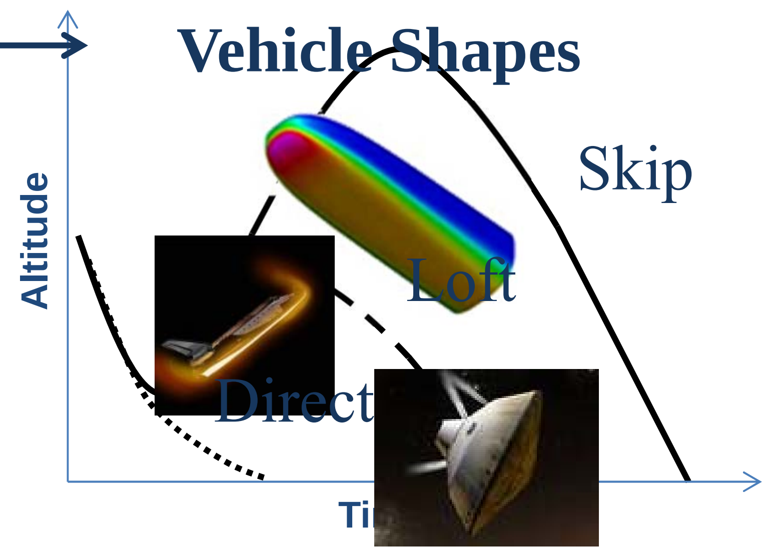
- Reduce the number of design cycles required to close an entry spacecraft design (potential cost savings)

Novel Research Objective

Develop a planetary guidance algorithm that is adaptable to:
-Mission Profiles
-Vehicle Shapes
for integration into vehicle optimization.

Mission Profiles

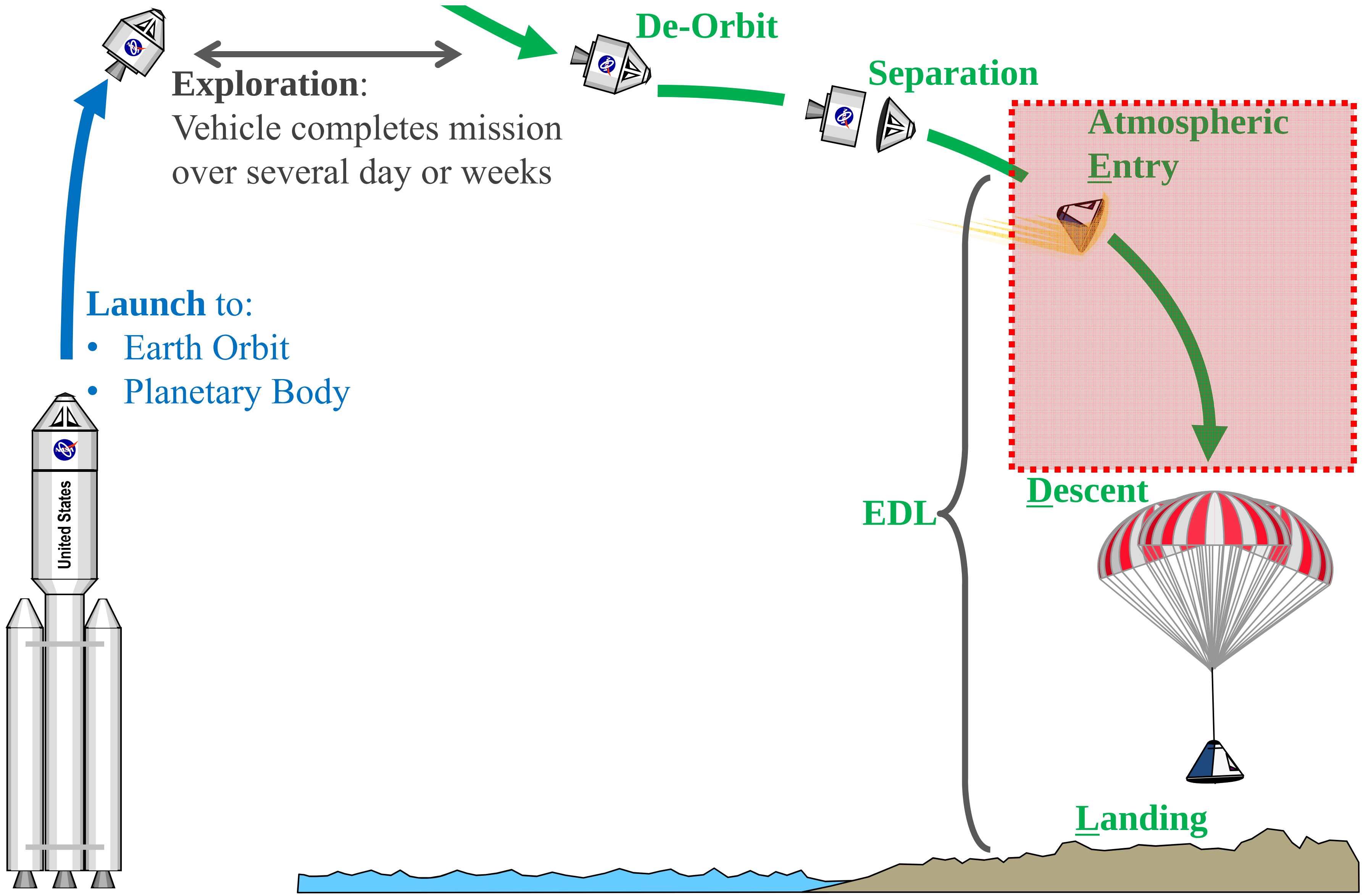
Vehicle Shapes



De-Orbit

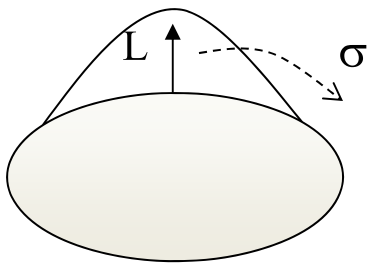
Sample Concept of Spaceflight Operations

* Adapted graphic from NASA Johnson Space Center



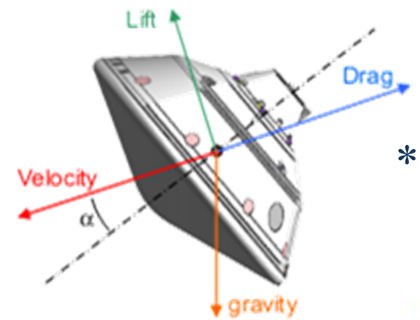
Planetary Entry Spacecraft Design (cont’d)

Mid - Low L/D Spacecraft



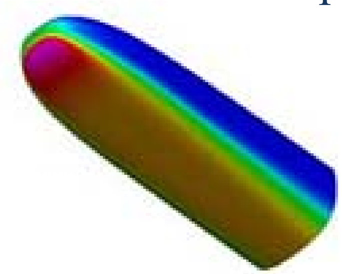
σ – variable bank angle
α – fixed angle of attack

* Orion Capsule
www.nasa.gov

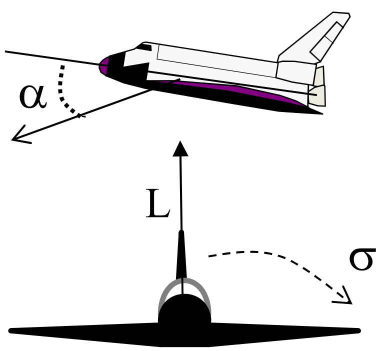


* MSL Capsule
Prakash et al.,
NASA JPL

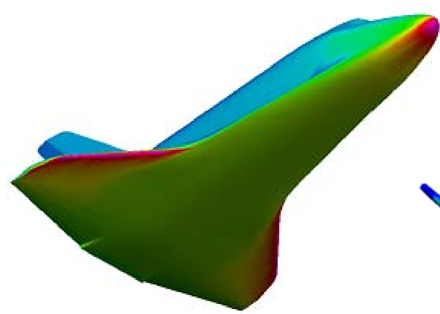
* Ellipsled
Garcia et al.,
AIAA Conf. Paper



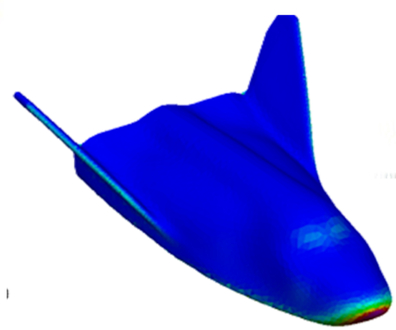
High L/D Spacecraft



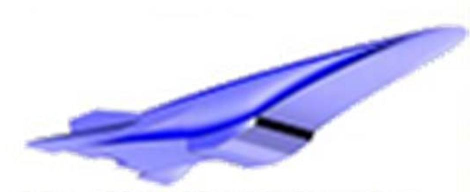
σ – variable bank angle
α – variable angle of attack



* Space Shuttle
AIAA 2006-659



* HL-20
AIAA 2006-239



* NASP
AIAA 2006-8013

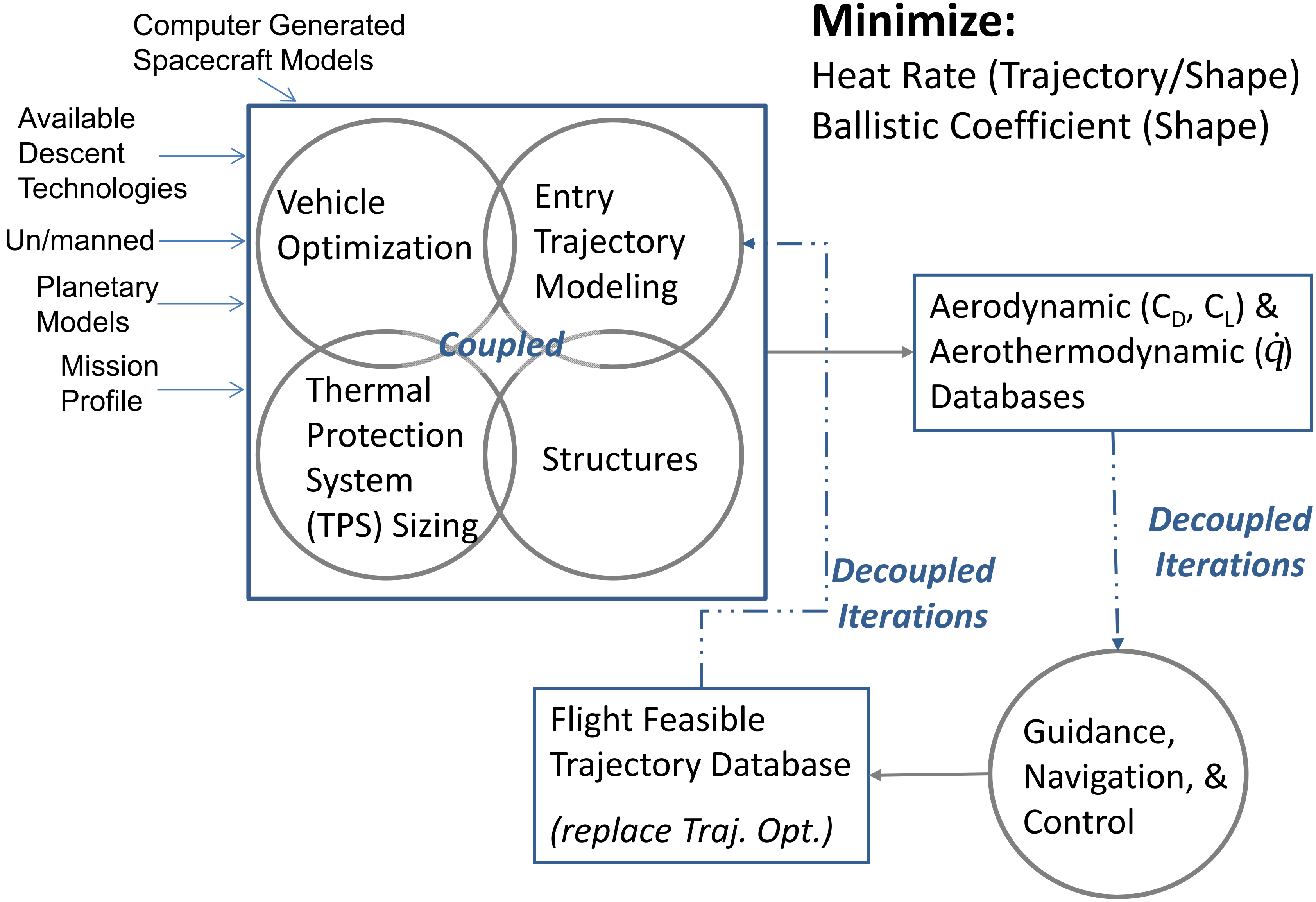
Multi-Disciplinary Design, Analysis, and Optimization

(MDAO)

Minimize:

Heat Rate (Trajectory/Shape)

Ballistic Coefficient (Shape)



Trajectory Optimization vs. Guidance

	Trajectory Optimization	**Guidance**
Constraints	Multiple included	Minimal included
Objective	Any variable of interest	Target specific
Solution	Purely numerical	Combination of numerical and analytical
Time to Solution	Minutes to hours	Seconds
Guaranteed Solution	No	Must enforce that a solution is found
Parameter Changes	Handles large parameter changes	Handles parameter changes that are relatively small
Result	**Nominal Trajectory – not always realistic control**	**Flight Feasible Trajectory with realistic controls**

Guidance Development Trade-Offs

Adaptability

Numerical formulation for adaptability to different vehicles and missions without significant changes

Rapid Trajectory Generation

Analytical driving function keep time to a solution low

Minimize Range Error & Heatload

Optimal Control theory to introduce heat load as an additional objective

Guidance Development Criteria

Guidance Specific (In-Flight)

- Determine flight feasible control vectors (control rate/acceleration constraints)
- Be highly robust to dispersions and perturbations
- Include a minimal number of mission dependent guidance parameters

Vehicle Design Specific

- Be applicable to multiple mission scenarios and vehicle dispersions
- Manage the entry heat load in addition to achieving a precision landing

Types of Guidance Techniques

Reference Tracking Only – follow a pre-defined track

In-flight Reference Generation & Tracking – Generate a real-time reference trajectory and follow that track

In-flight Controls Search – One dimensional search, usually solving equations of motion numerically

In-flight Optimal Control – Requires numerical methods to meet some cost function

Types of Guidance Formulations

	Analytical Guidance	Numerical Guidance
Advantages	• Simple to Implement • Computation time minimal • Solution Guaranteed	• Accurate trajectory solutions • No simplifying assumptions (possibility of multiple entry cases to be simulated with few modifications)
Disadvantages	• Simplifications reduce accuracy of the trajectory solution • Formulation tied to a specific entry case	• Convergence is not assured • Convergence is not timely

Novel Approach to Guidance for MDAO

Real-Time Trajectory Generation and Tracking

Adaptability

Numerically solve entry equations of motion

Use generalized analytical functions to represent the reference

Adaptation of Shuttle Entry Guidance Techniques

Rapid Trajectory Generation

Use analytical driving function keep time to a solution low

Use Single Optimal Control Point with Blending

Adaptation of Energy State Approximation Techniques

Minimize Range Error & Heatload

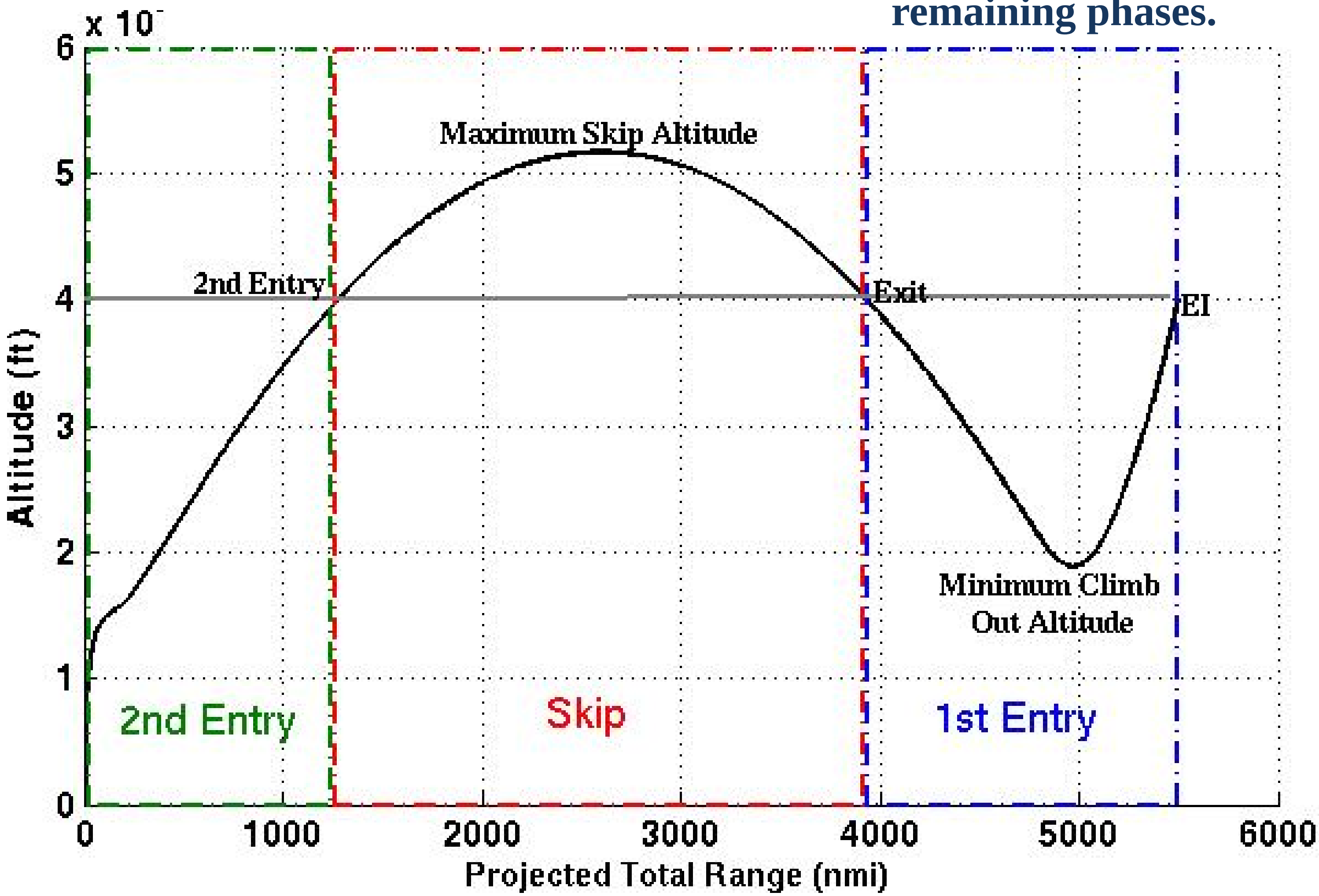
Optimal Control theory used to introduce heat load objective

Skip Entry Critical Points

Test Case: Orion Capsule, L/D 0.4

Control: Bank Angle only

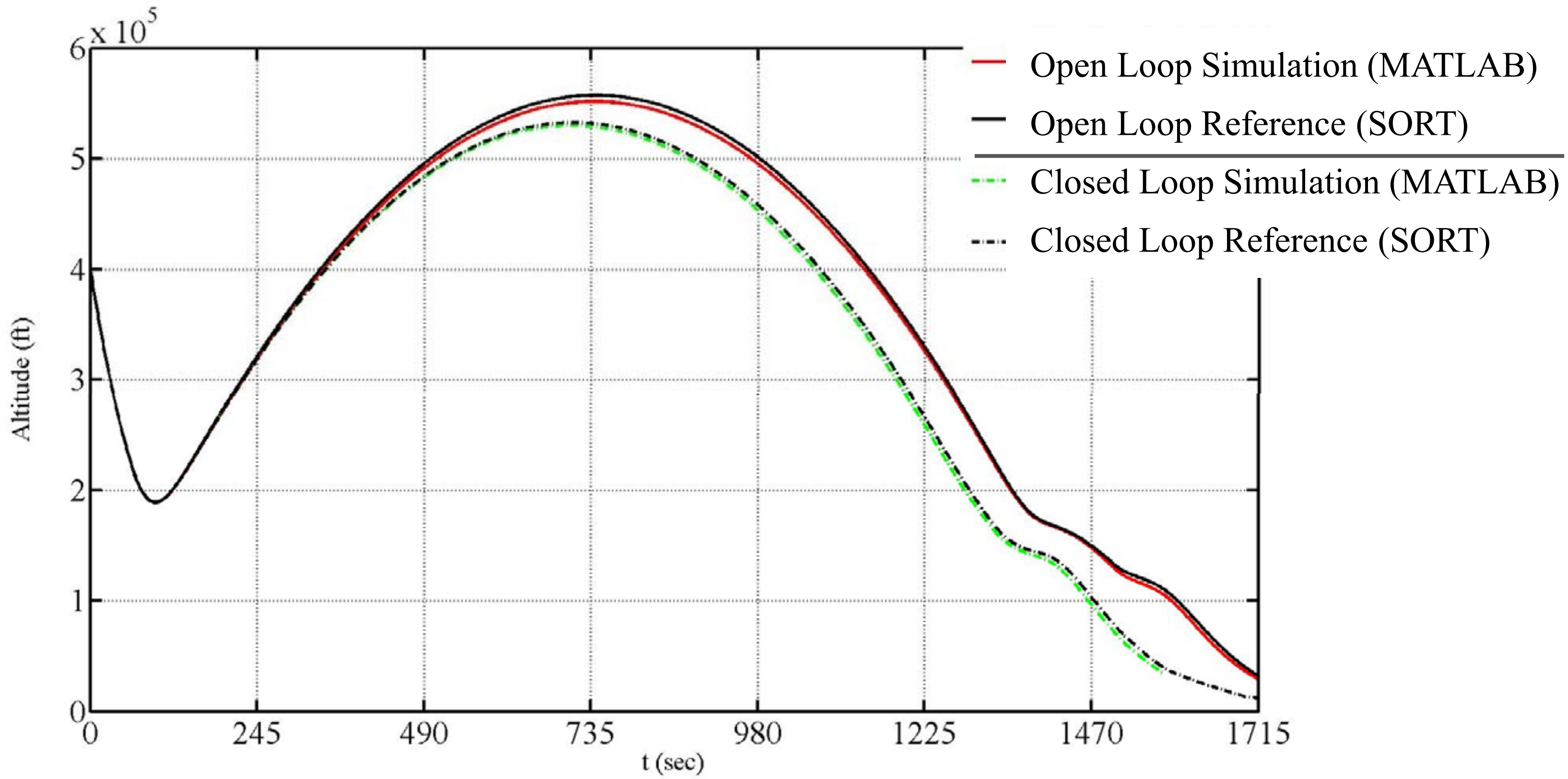
Begin with 1st Entry portion of the trajectory and gradually includes remaining phases.



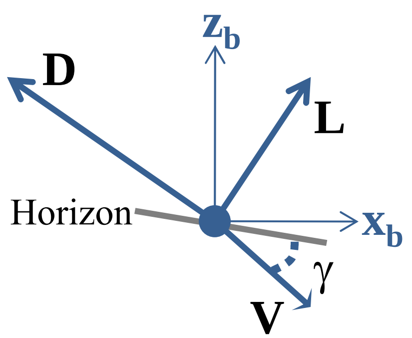
Trajectory Simulation Validation

Truth Model

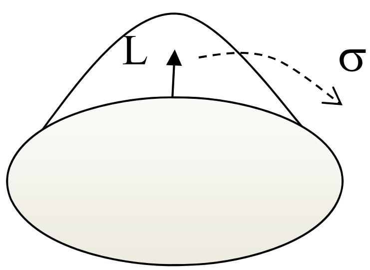
Simulation of Rocket Trajectories (SORT)
Developed by NASA Johnson Space Center for Space Shuttle Launch/Entry Simulations



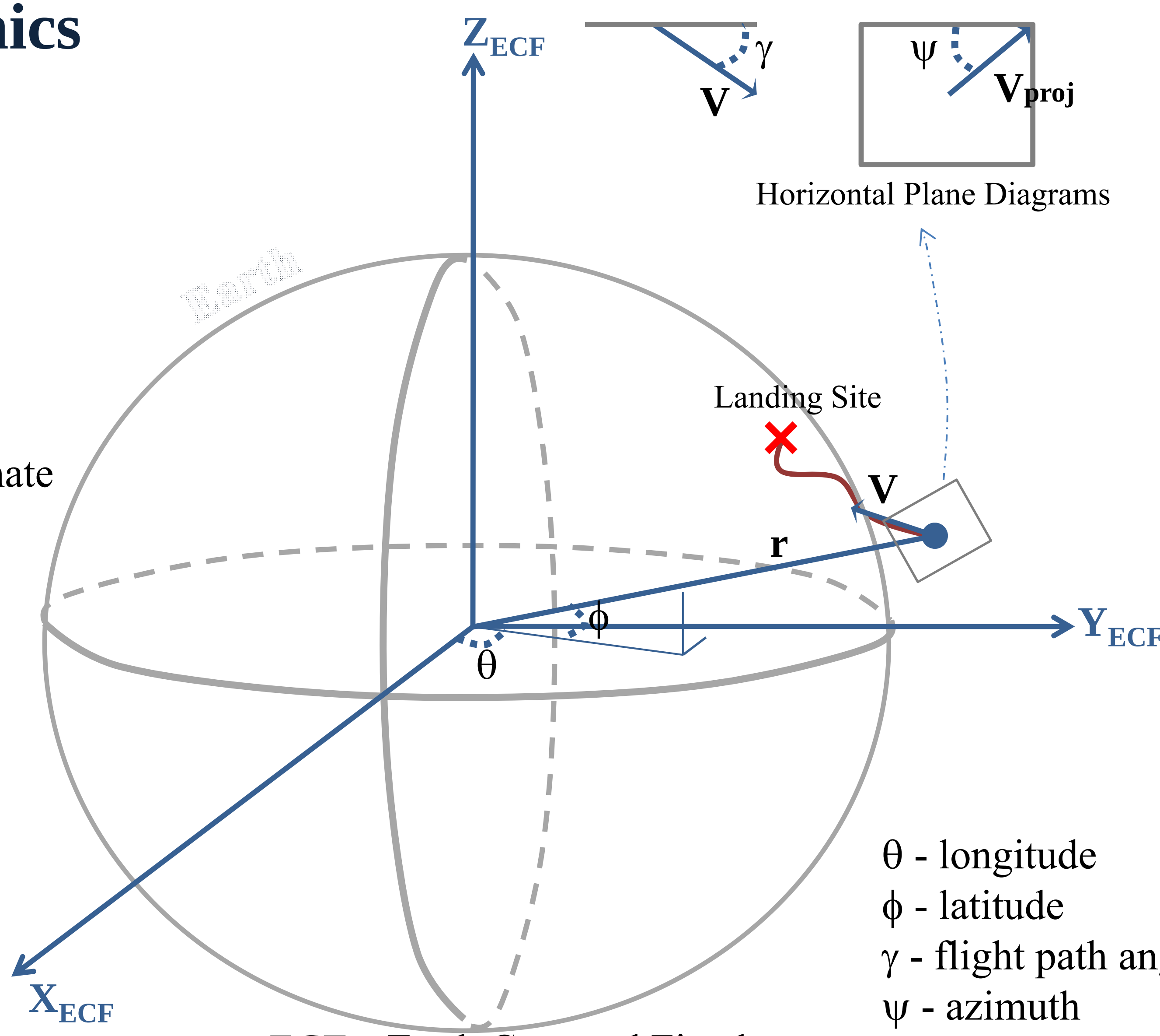
Flight Dynamics



b – body fixed coordinate



σ – bank angle



ECF – Earth Centered Fixed

θ - longitude
ϕ - latitude
γ - flight path angle
ψ - azimuth

Trajectory Modeling

$$\dot{r} = V \sin\gamma$$

$$\dot{\theta} = \frac{V\cos\gamma\sin\psi}{r\cos\phi}$$

$$\dot{\phi} = \frac{V\cos\gamma\cos\psi}{r}$$

$$\dot{V} = -D - g\sin\gamma + \Omega^2 r\cos\phi(\sin\gamma\cos\phi - \cos\gamma\sin\phi\cos\psi)$$

$$\dot{\gamma} = \frac{1}{V}\left[L\cos\sigma + \cos\gamma\left(\frac{V^2}{r} - g\right) + 2\Omega V\cos\phi\sin\psi + \Omega^2 r\cos\phi(\cos\gamma\cos\phi + \sin\gamma\cos\psi\sin\phi)\right]$$

$$\dot{\psi} = \frac{1}{V}\left[\frac{L\sin\sigma}{\cos\gamma} + \frac{V^2}{r}\cos\gamma\sin\psi\tan\phi - 2\Omega V(\tan\gamma\cos\psi\cos\phi - \sin\phi) + \frac{r\Omega^2}{\cos\gamma}\sin\psi\sin\phi\cos\phi\right]$$

State Variables

r - radial distance

V - relative velocity

θ - longitude

ϕ - latitude

γ - flight path angle

ψ - azimuth

Control Variables

σ - bank angle

α - angle of attack

Vehicle and Planet Variables

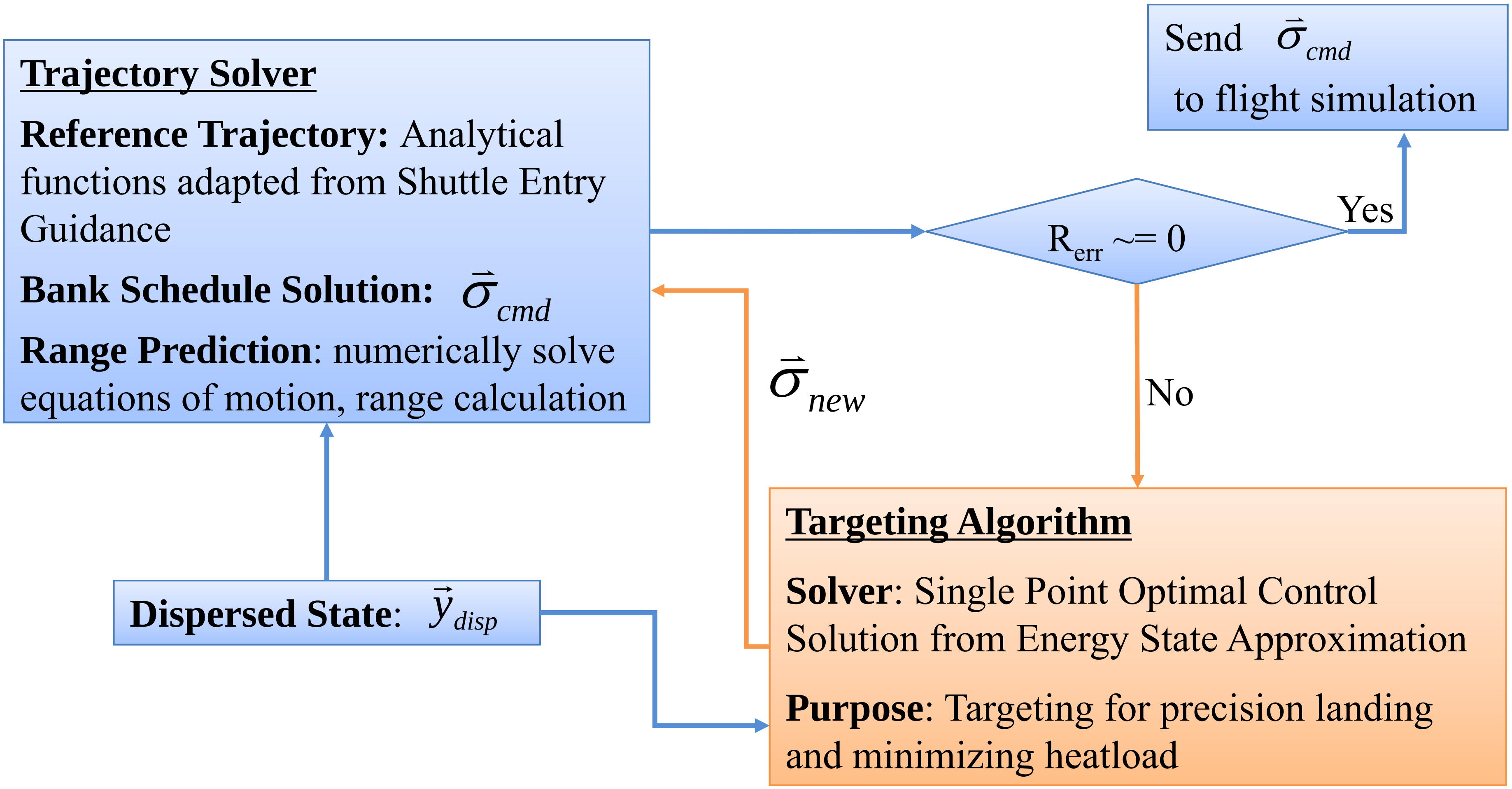
L, D - Lift, Drag Acceleration

g - gravity

Ω - Earth ‘s Rotation

ρ – atmospheric density

General Entry Guidance Block Diagram



Control Solution: Shuttle Entry Guidance Adaptation

Shuttle Entry Guidance (SEG) Concept: Temperature Phase

- Reference Tracking Algorithm, Closed Form Solution

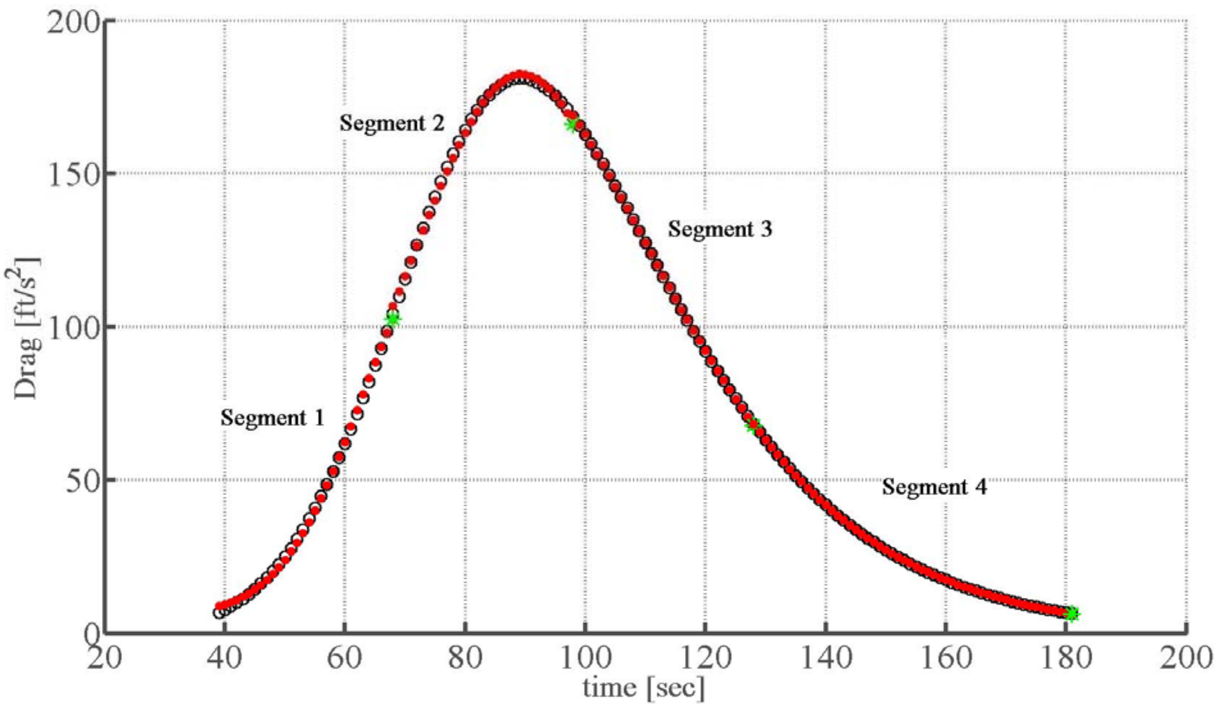
$$\frac{d}{dt}\left(D = \frac{\rho V_r^2 C_D A}{2m}\right) \longrightarrow \dot{\rho} = \frac{d}{dt}\left(\rho_o e^{\frac{-h}{h_s}}\right) \Rightarrow \frac{\dot{\rho}}{\rho} = -\frac{\dot{h}}{h_s}$$

Reference Trajectory

$$D_{ref} = C_2 V^2 + C_1 V + C_0$$

$$\gamma_{ref} = \text{constant}$$

Bank Schedule ... ***tion***



Control Solution: Shuttle Entry Guidance Adaptation

Improvements on Shuttle Entry Guidance "Drag Based Approach"

•Increase # of segments

•Increase order of polynomial

•Change Atmospheric Model representation

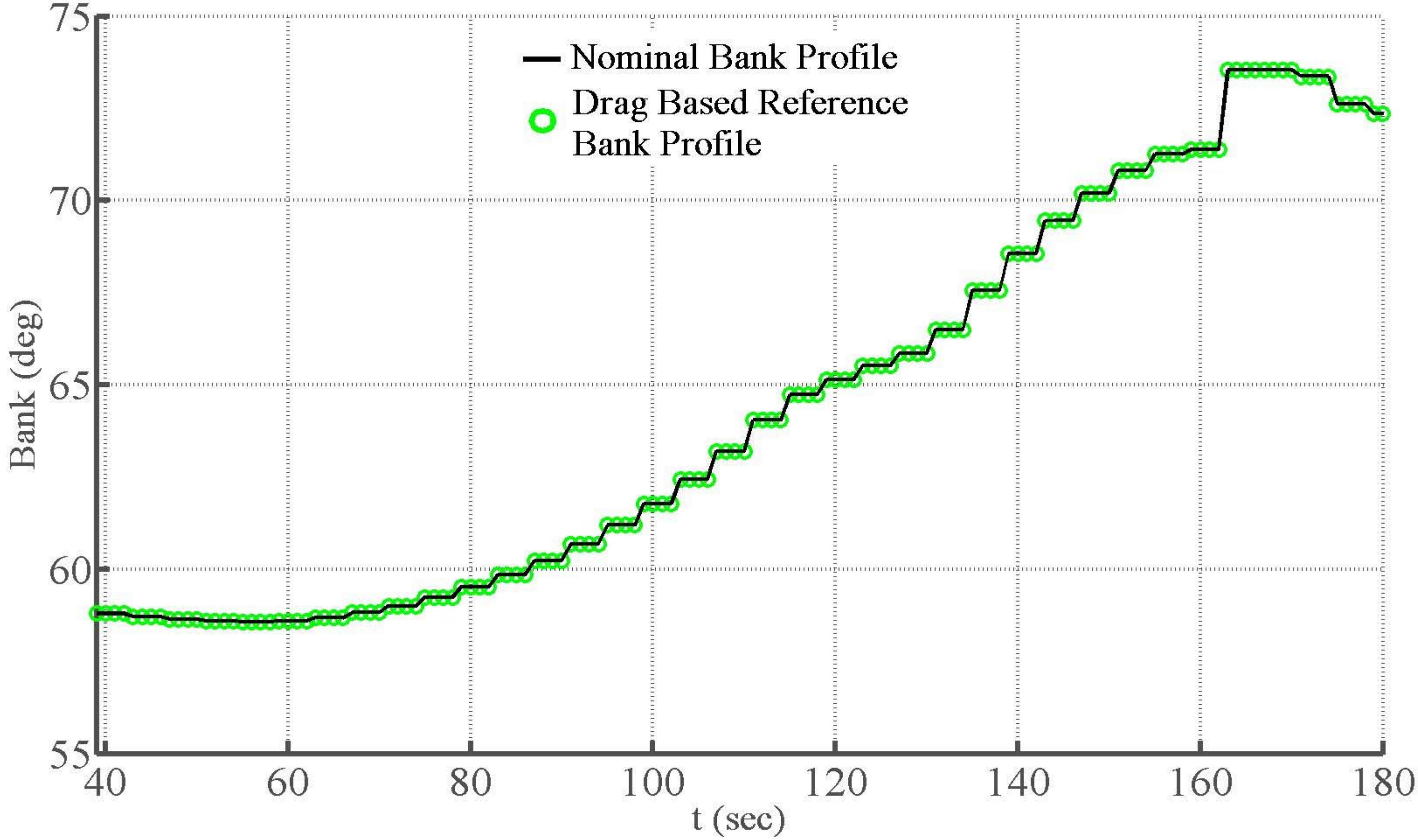
•Modify flight path angle representation

Challenges with Drag Based Approach

- Discontinuities between segments
- Increasing # of coefficients for storage with increasing segments and/or order
- Effect of small flight path angle assumption unknown
- Formulations are derived from 2DOF Longitudinal EOMs

Control Module: Shuttle Entry Guidance Adaptation

Sensitivity to atmospheric non-linearity is significant during initial and final segments. **Need an Alternative Analytical Equation!**



Automated Selection of Transition Events

Framework:

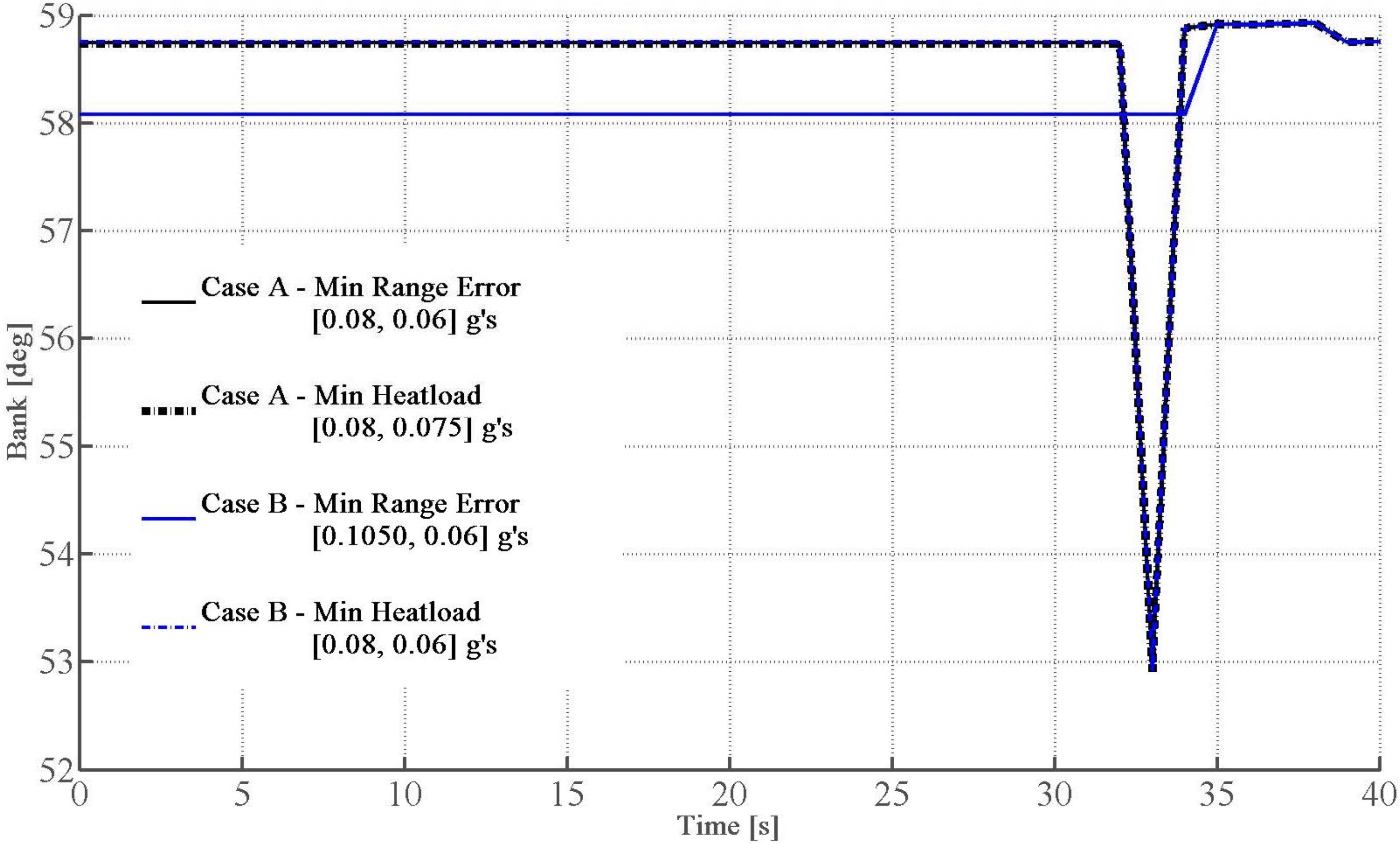
- Allows for adaptability
- Automated generation of Reference Trajectory
- Open loop

Study Objective: Define bank profile for trajectory phases

Phase	Bank Description
Entry Interface to Guidance Start	Constant Bank
Guidance Start to Guidance End	Trajectory Solver
Guidance End to Exit	Linear Transition to Meet 2^{nd} Entry Bank
Exit to 2^{nd} Entry	Attitude Hold

Automated Selection of Transition Events

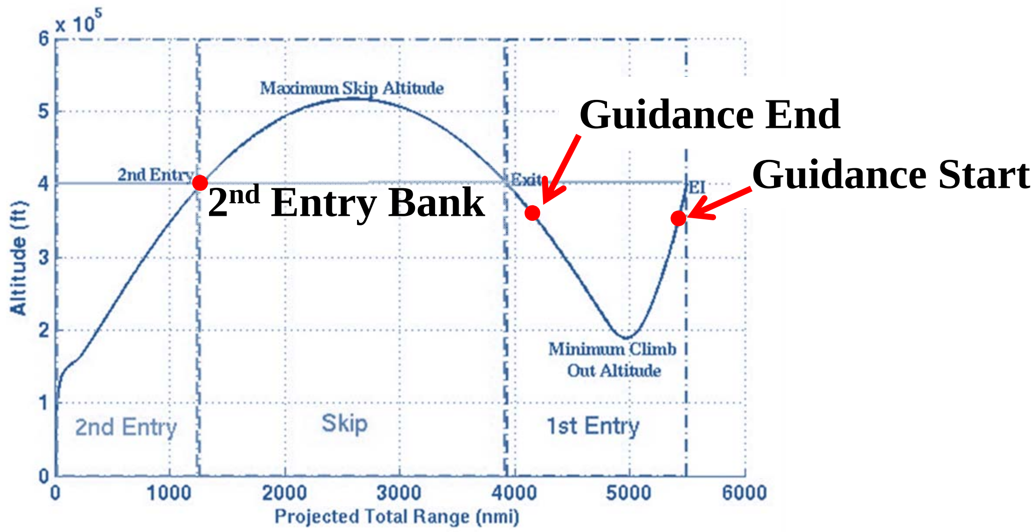
- Metric to determine best trajectory: lowest range error, lowest heat load from EI to 2nd Entry, and bank transitions



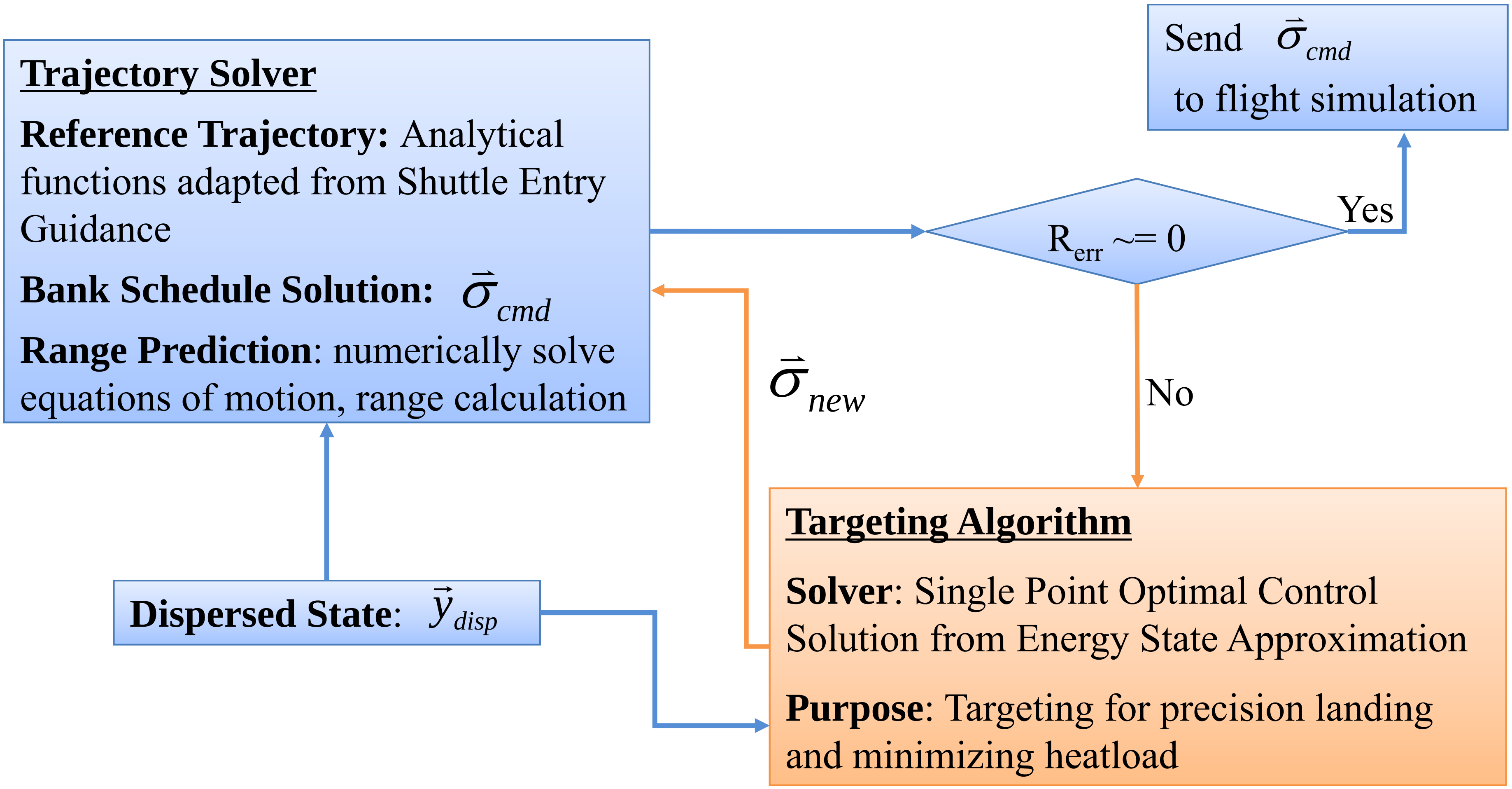
Automated Selection of Transition Events

Study Results:

Phase	Bank Description
Entry Interface to Guidance Start	Constant Bank = **57.95°**
Guidance Start to Guidance End	Trajectory Solver **{0.12 0.11} G's**
Guidance End to Exit	Linear Transition to Meet 2nd Entry Bank Linear Transition Velocity: **23,784.65 ft/s**
Exit to 2nd Entry	Bank Attitude Hold = **70°**



General Entry Guidance Block Diagram



Targeting Algorithm Development

When is Targeting Activated?

1. Overshoot – Vehicle is predicted to fly <u>way past target</u>
2. Undershoot – Vehicle is predicted to fly <u>short of the target</u>

How to find a set of controls to Correct Over/Underhoot?

Adapt Energy State Approximation Methods:
Optimal control method that replaces altitude and velocity with specific energy height (e)

$$e = \frac{V_r^2}{2g_o} + h$$

Advantages: Allows for a compact set of analytical equations

Add heat load to the range error objective function

Disadvantage: Optimal control formulations may not converge to a solution

Solution: Derive a **<u>localized</u>** optimal control point instead and blend back reference trajectory

Targeting Algorithm Development

Must Relate Euler-Lagrange Equation

$$\bar{\lambda} = \frac{\lambda_\psi}{\lambda_\gamma} = \tan\sigma^* \cos\gamma$$

$$\lambda\gamma \leq 0$$

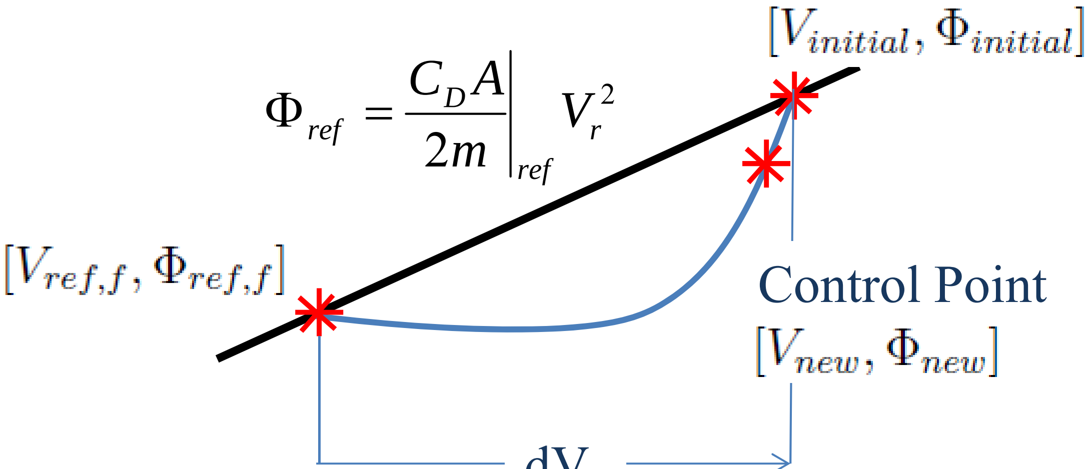
To Reference Trajectory Variables

$$\left.\frac{L}{D}\right|_{total} \cos\sigma = \frac{1}{\rho\Phi_{ref}}\left[V_r\dot{\gamma}_{ref} - \cos\gamma\left(\frac{V_r^2}{r} - g\right) - C_\gamma(y)\right]$$

Using trigonometry and other manipulations, the control equation is found

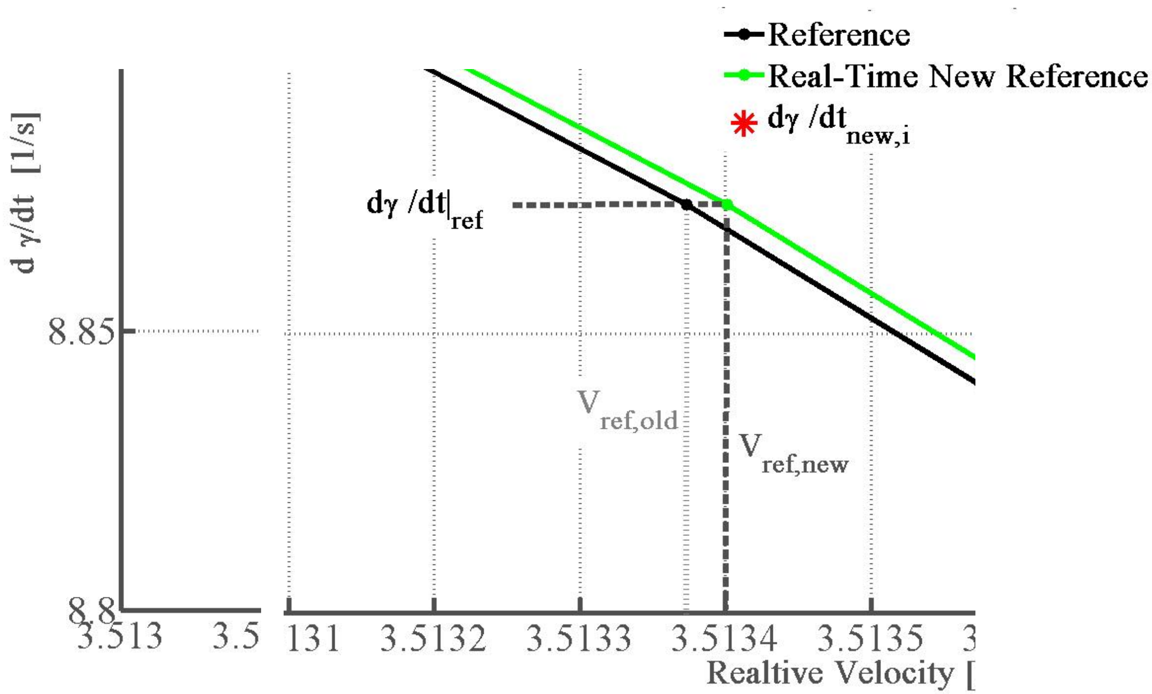
$$\left.\frac{L}{D}\right|_{total}\sqrt{\frac{1}{1+\left(\frac{\bar{\lambda}}{\cos\gamma}\right)^2}} = \frac{\left[V\dot{\gamma_{ref}} - \cos\gamma\left(\frac{V^2}{r} - g\right) - 2\Omega V\cos\phi\sin\psi - \Omega^2 r\cos\phi(\cos\gamma\cos\phi + \sin\gamma\cos\psi\sin\phi)\right]}{D_{aprx}}$$

Targeting Algorithm Development



Least Squares Curve Fitting:
3 Interpolation Points

$$\Phi_{blnd} = Bb_2V^2 + Bb_1V + Bb_0$$



Targeting Algorithm Development

Targeting Technique 1 – Design Space Interrogation

C_Φ - drag/density ratio coefficient

$d\lambda$ - change in Lagrange multiplier

dV - change in relative velocity at next point

Targeting Technique 2 – Design Space Interrogation

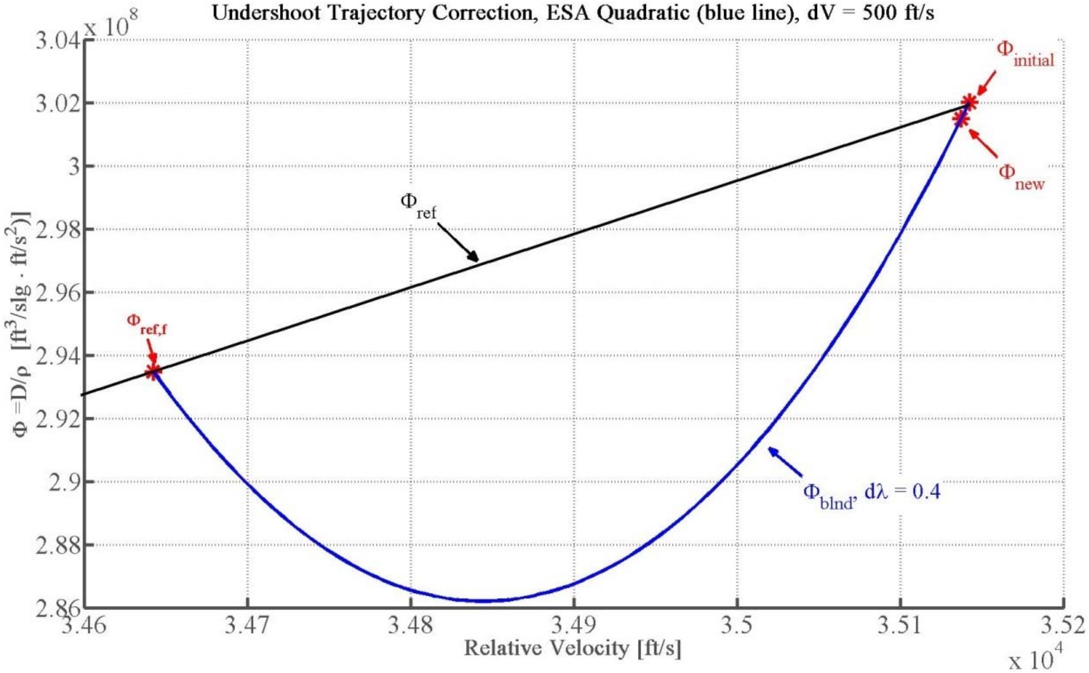
$d\lambda$ - change in Lagrange multiplier

dV_1 - change in relative velocity halfway to curve fit end point

$\Delta(dE)$ - second order change in energy

Targeting Algorithm Development

Targeting Technique 1 – Design Space Interrogation

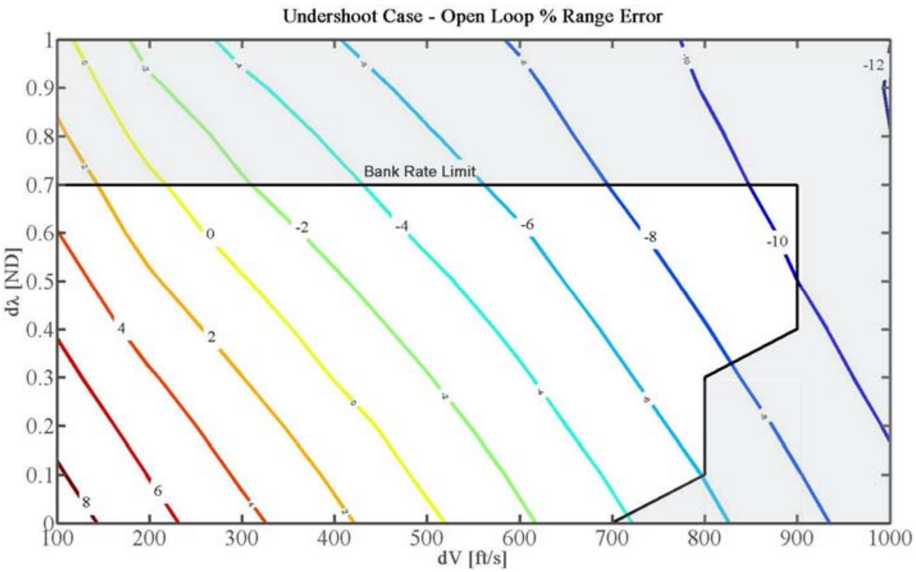


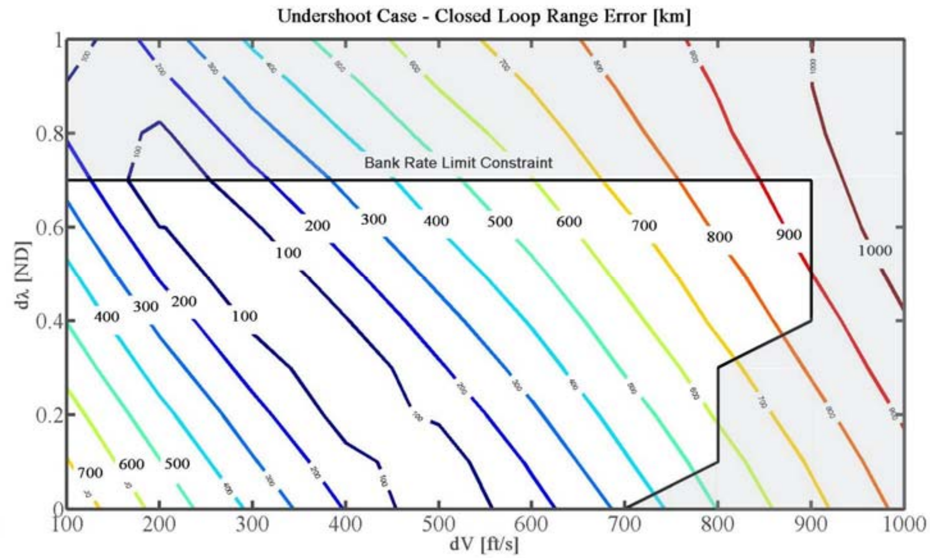
	Lower Limit	*Upper Limit*	*Incr.*	*units*
C_Φ	0	1		ND
$d\lambda$	0	1	0.01	ND
dV	100	1000	100	ft/s

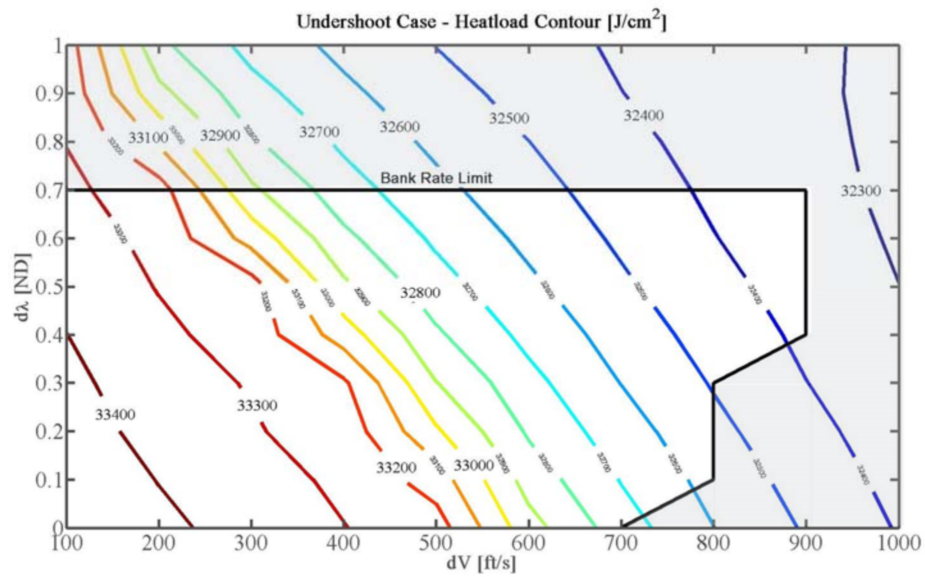
Case	*Dispersion*	*Target Miss*
1	Increase Entry Flight Path Angle	Undershoot
2	Decrease Entry Flight Path Angle	Overshoot
3	L/D Dispersion	Overshoot

Targeting Algorithm Development

FPA Dispersion - Undershoot

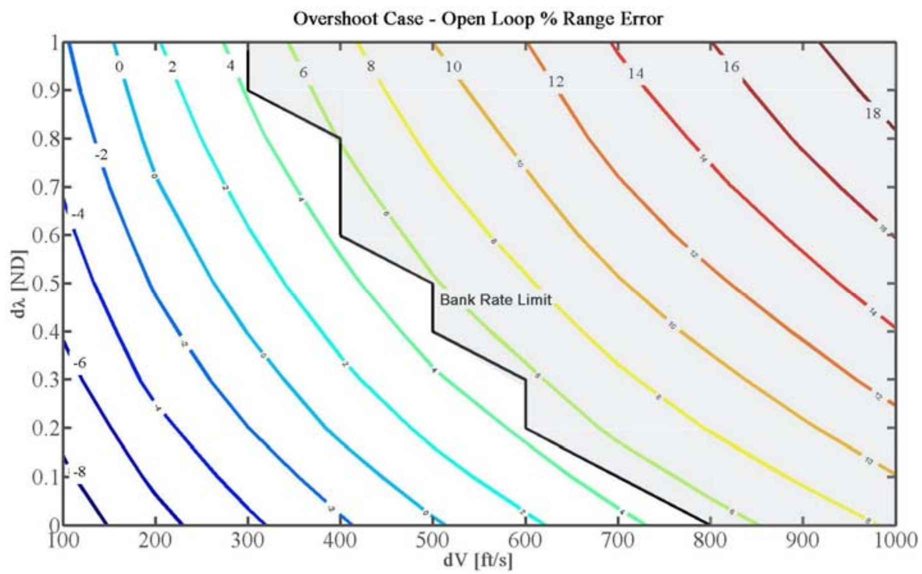


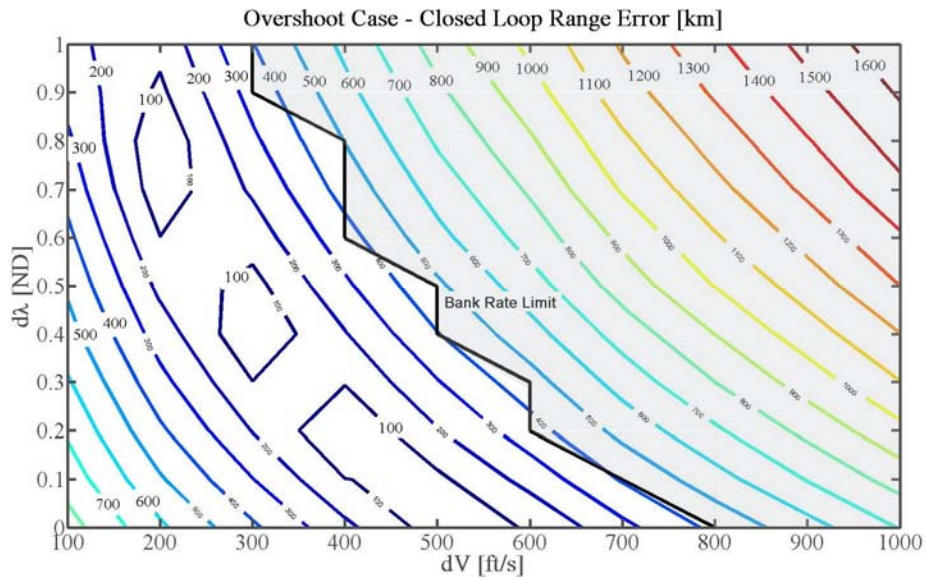


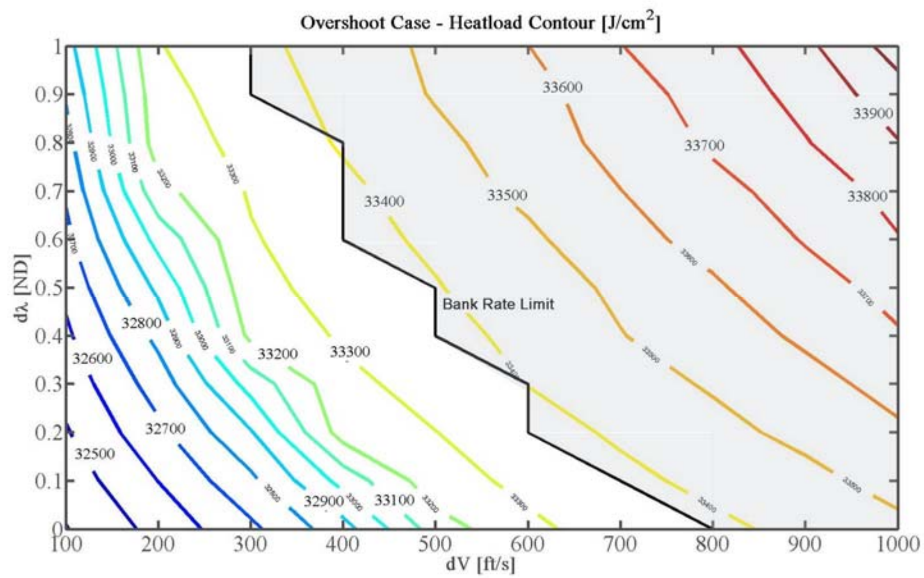


Targeting Algorithm Development

FPA Dispersion - Overshoot

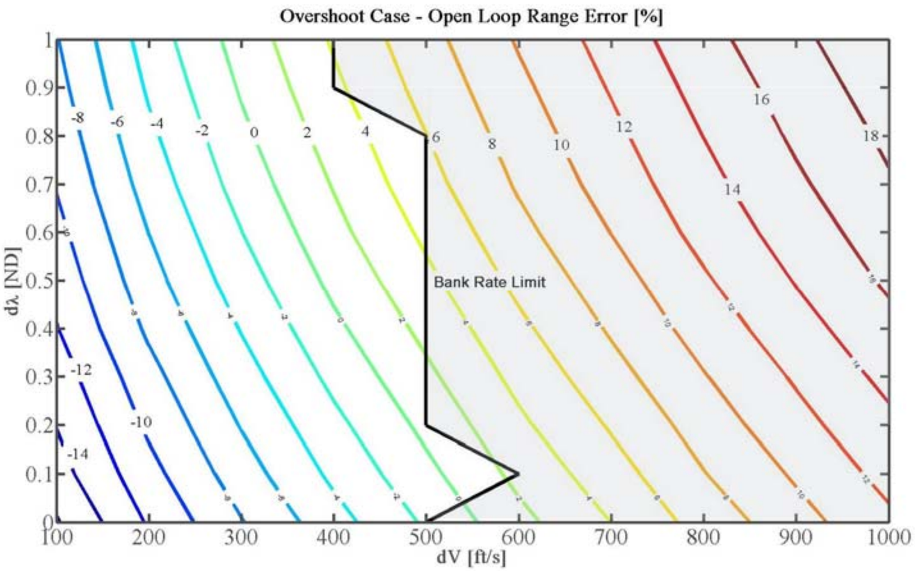


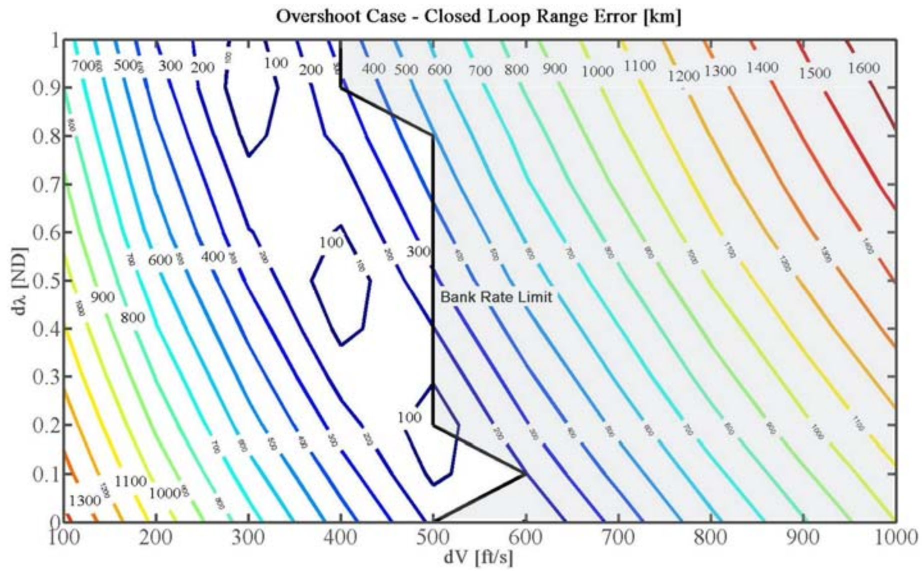


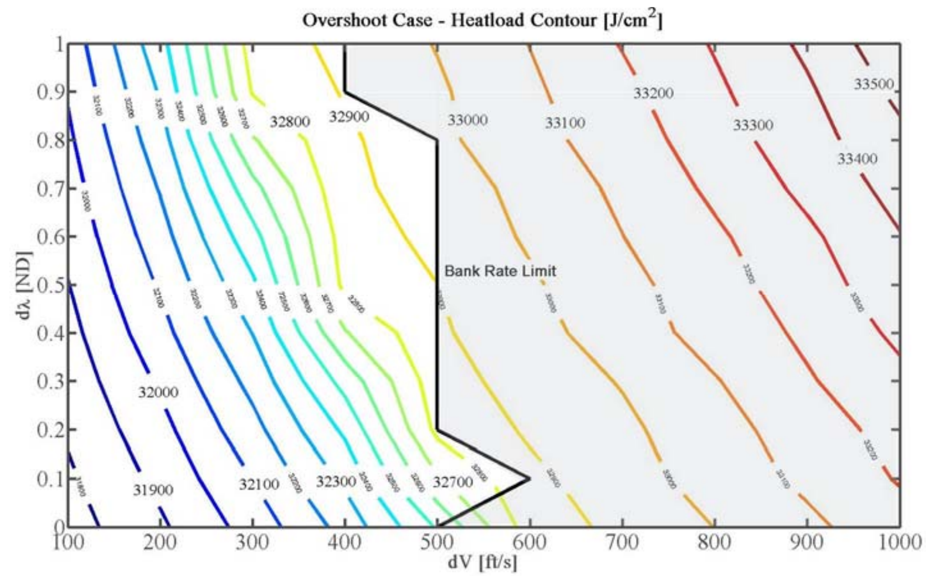


Targeting Algorithm Development

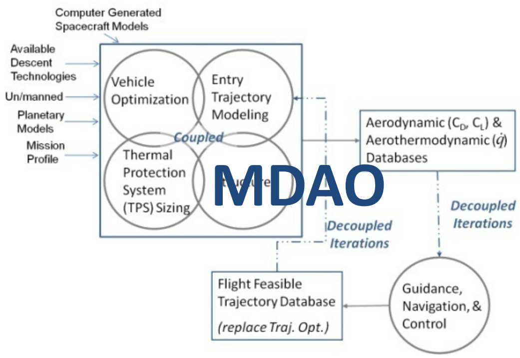
Aerodynamic Dispersion - Overshoot



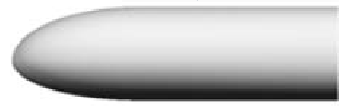




Shape Optimization Analog



Geometry #1: C_L = 1.70, C_D = 3.4



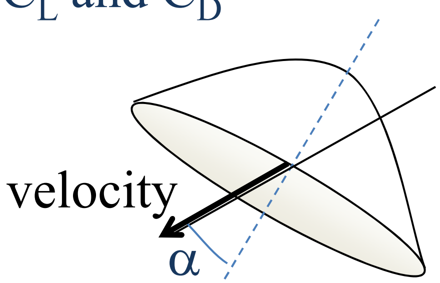
Geometry #2: C_L = 1.90, C_D = 3.8

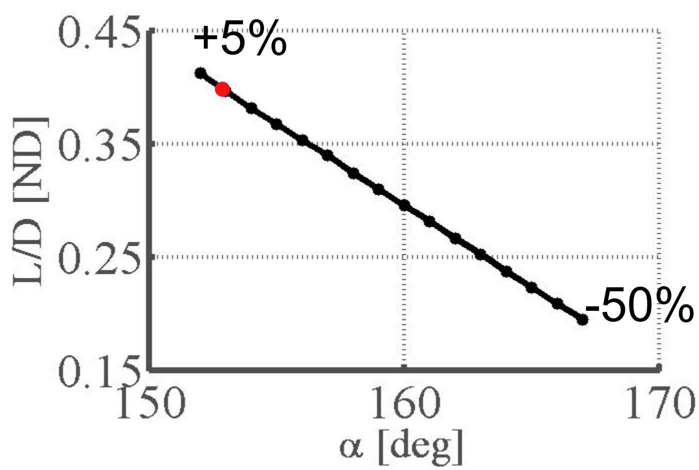
Geometry #3: C_L = 1.95, C_D = 3.9

Current Guidance Algorithms – Robust to ~20% aerodynamic dispersions

Must exceed 20% to demonstrate potential for integration into MDAO

ANALOG: Changing angle of attack disperses C_L and C_D



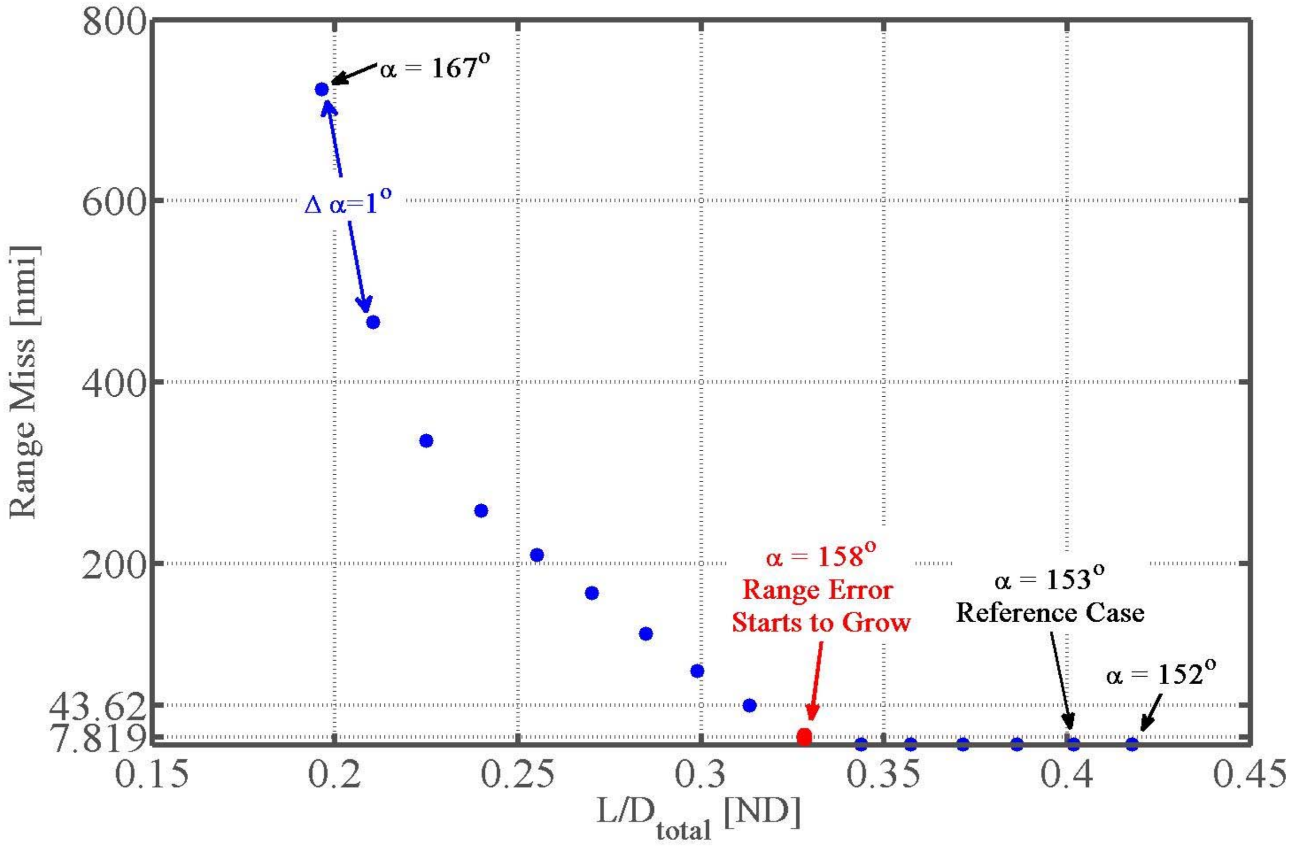


Targeting Algorithm Development

Guidance Algorithm for Comparison – Apollo Derived Final Phase Guidance

Reference Tracking to a stored trajectory database, function of relative velocity

Performance Results – Threshold Miss Distance, 1 nmi



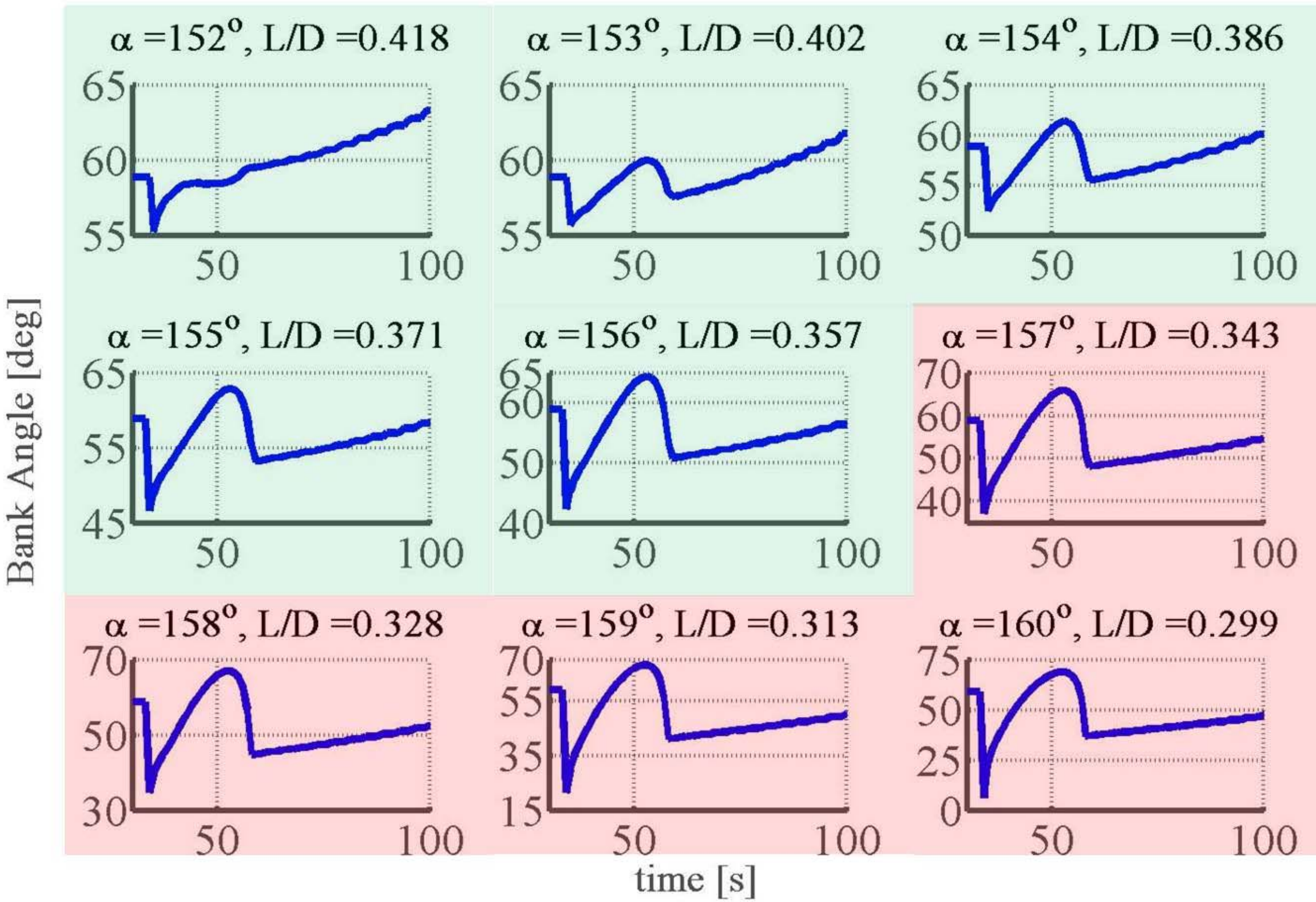
Targeting Algorithm Development

Targeting Technique 1 – Targeting Procedure

1. Guess a value for $d\lambda$
2. Iterate on dV using secant method to converge on a zero range error trajectory
3. If no solution is found, $d\lambda$ is incremented and the iteration is repeated
4. Solution is then flown in flight simulation

Targeting Algorithm Development

Targeting Implementation, 1st and 2nd Phase - Results



Targeting Algorithm Development

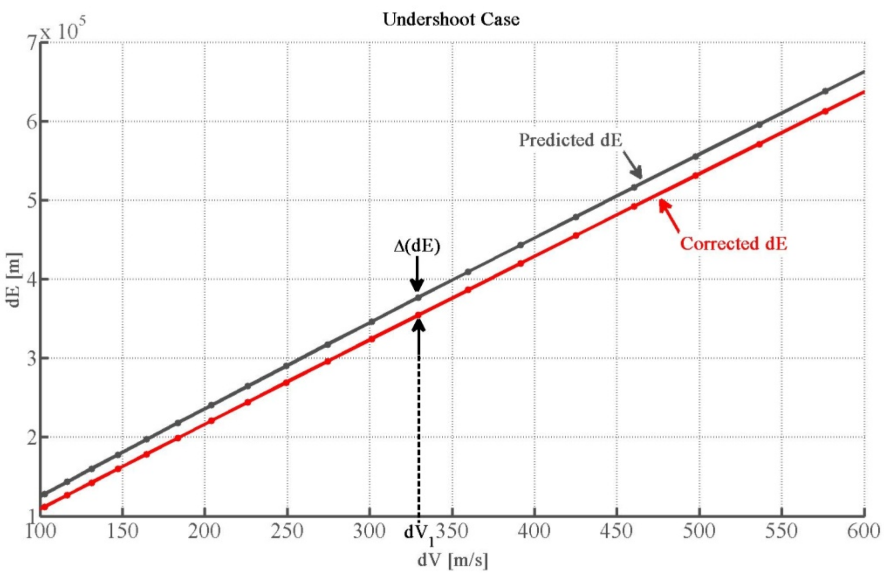
Targeting Technique 2

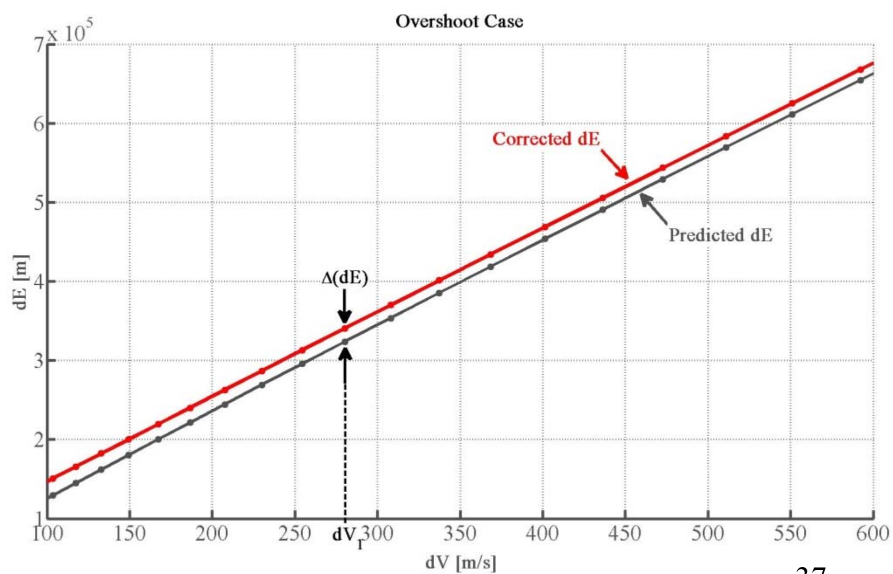
Use Energy Height $e = \frac{V_r^2}{2g_o} + h$ to determine Control Point $[V_{new}, \Phi_{new}]$

Undershoot → energy dissipating (de/dt) too fast

Overshoot → energy dissipating (de/dt) too slow

Since Velocity is an independent variable
and a pseudo control de/dV is examined



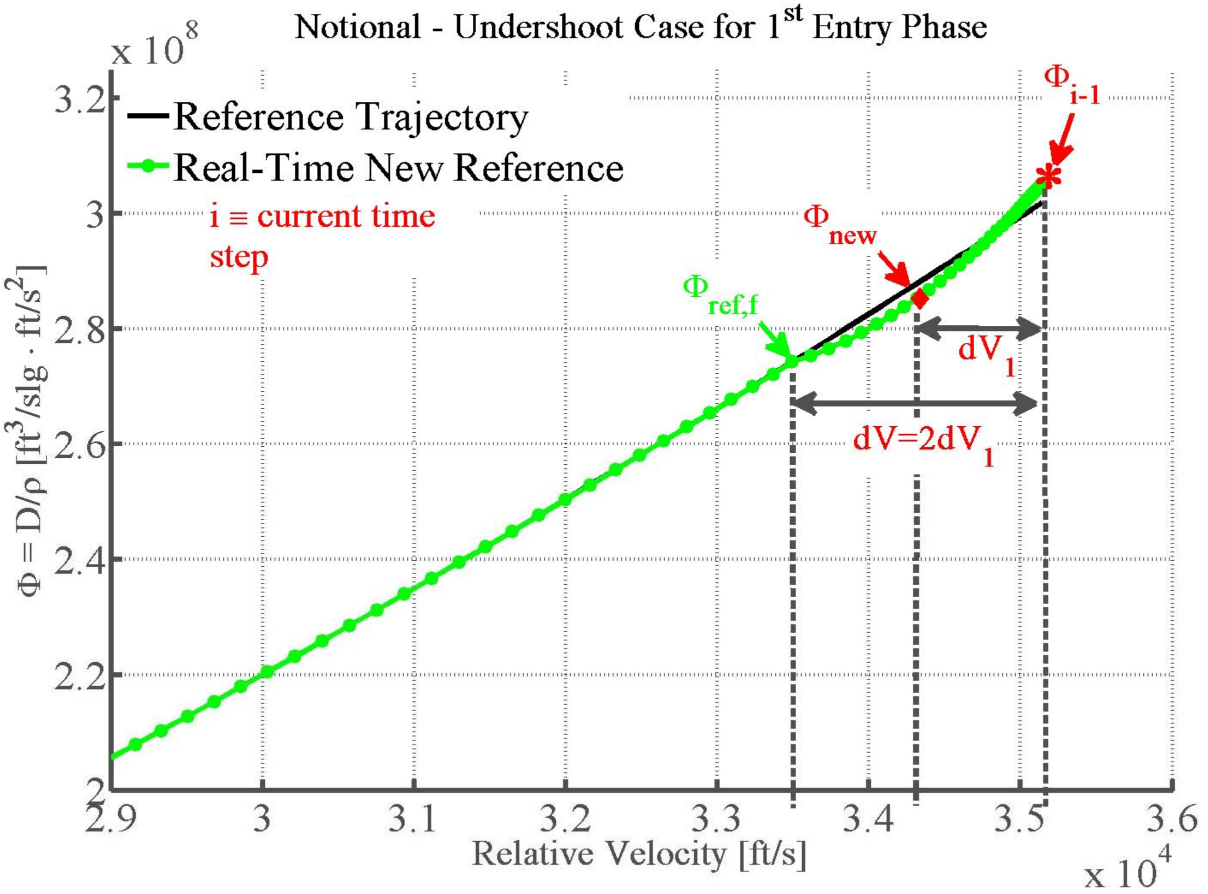


Targeting Algorithm Development

Targeting Technique 2

Recall the equation for the ratio of drag acceleration to density: $\left.\frac{D}{\rho}\right| = \frac{C_D A}{2m} V_r^2$

-Extract altitude and velocity from $[dV_1, \Delta(dE)]$ to find Φ_{new}



Targeting Algorithm Development

Targeting Technique 2 – Design Space Interrogation

	Lower Limit	*Upper Limit*	*Incr.*	*units*
$d\lambda$	0	$d\lambda_{limit}$		ND
dV_1	0	1524	Predict	m/s
$\Delta(dE)$	0	$\Delta(dE)_{limit}$	Predict	m

Limit are trajectory dependent
and control system dependent

$$\bar{\lambda}_{min/max} = \tan\sigma_{min/max}\cos\gamma_i$$

$$d\lambda_{limit} = \mp\left(\bar{\lambda}_{min/max} - \bar{\lambda}_{old}\right)$$

<u>Dispersion Cases:</u>

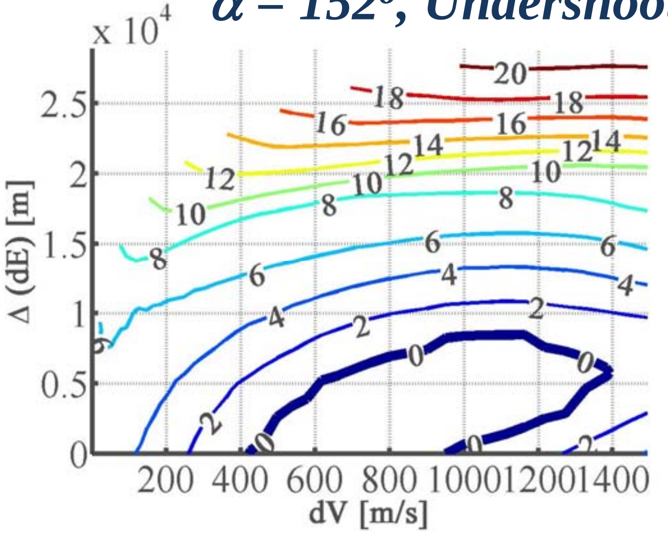
1st Phase Only

α [deg]	*L/D Dispersion*	*Target Miss*
Nominal	0.4 (0%)	
152	0.42 (+5%)	Undershoot
162	0.28 (-30%)	Overshoot
165	0.23 (-43%)	Overshoot
167	0.2 (-50%)	Undershoot

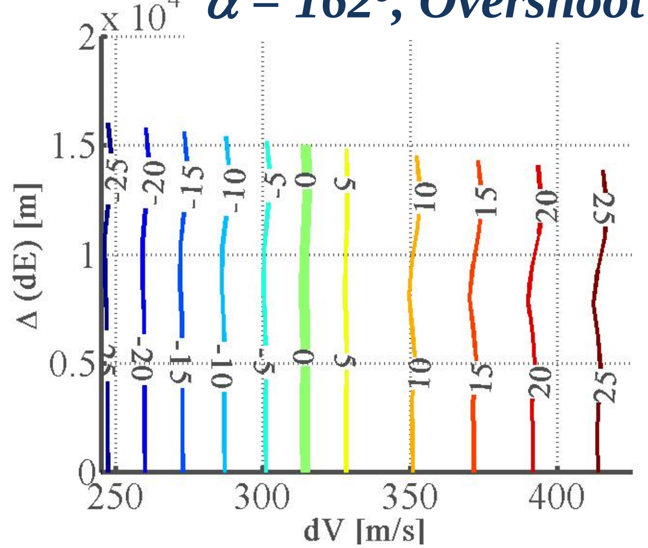
Targeting Algorithm Development

Design Space Interrogation, Results: *Range Error [%]*

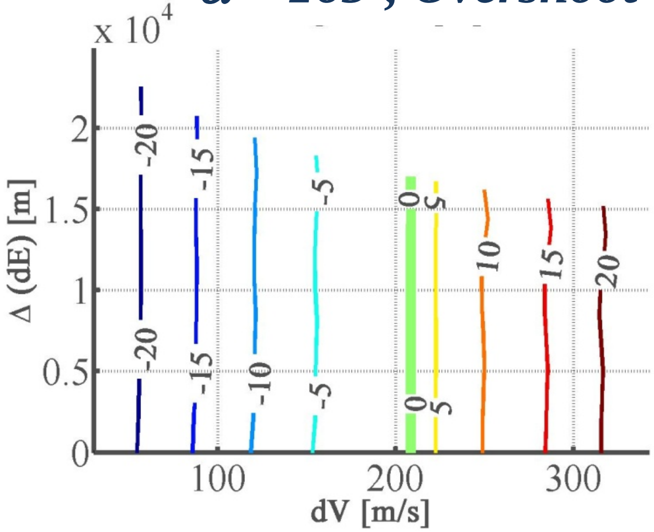
α = 152º, Undershoot



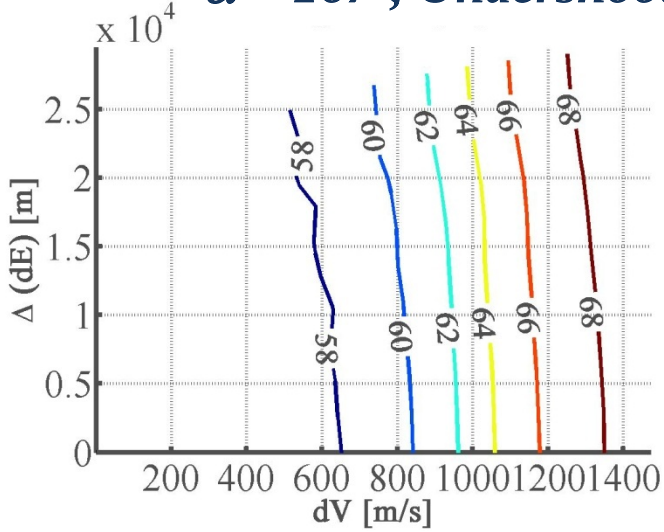
α = 162º, Overshoot



α = 165º, Overshoot



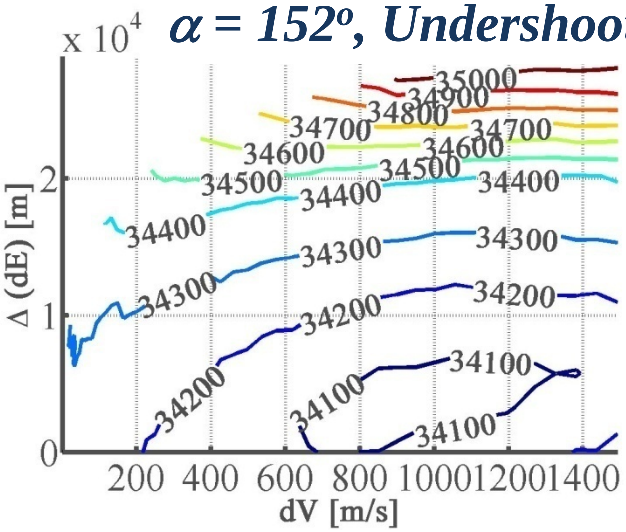
α = 167º, Undershoot



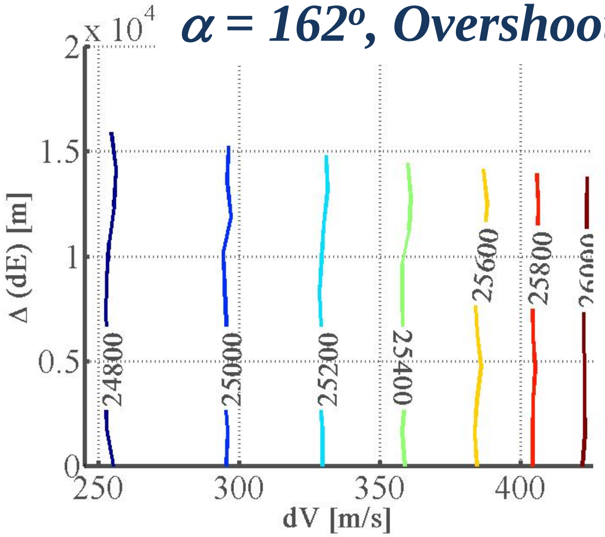
Targeting Algorithm Development

Design Space Interrogation, Results: *Heatload [J/cm^2]*

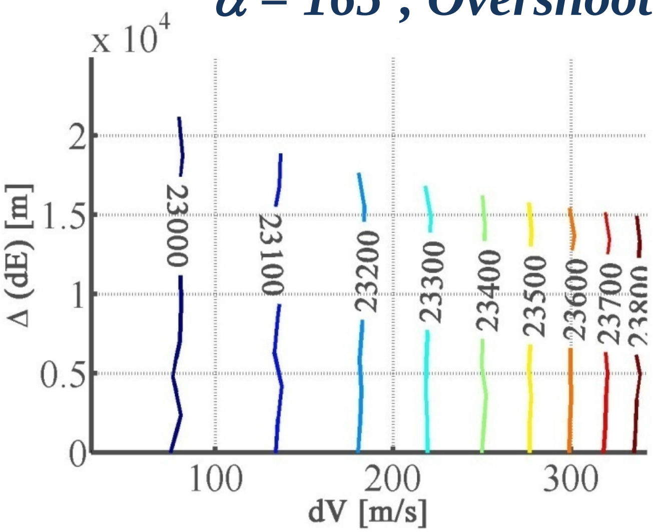
α = 152º, Undershoot



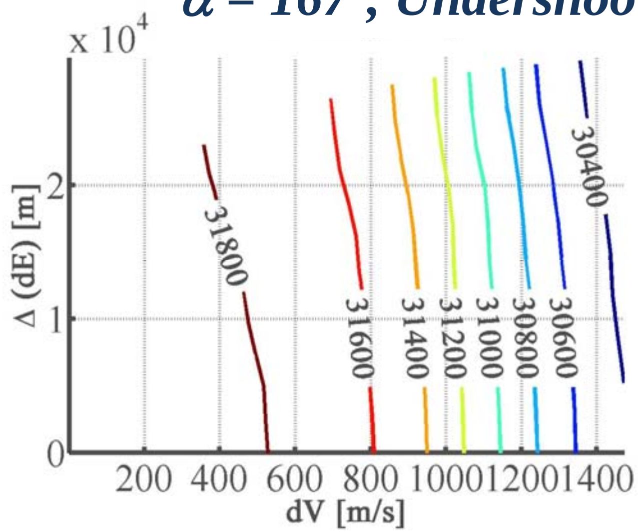
α = 162º, Overshoot



α = 165º, Overshoot



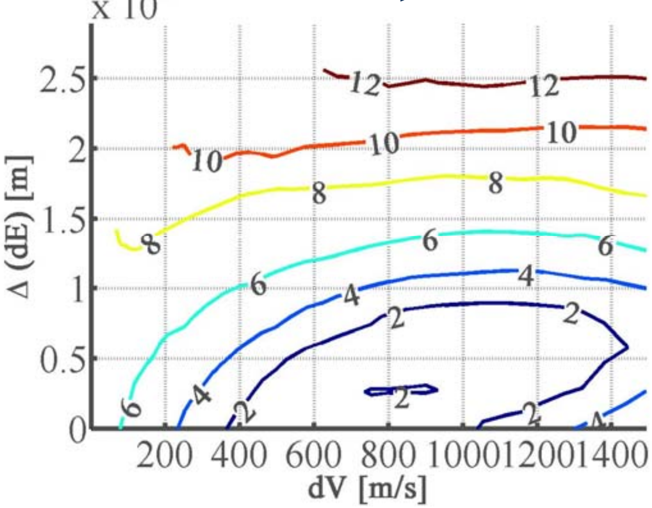
α = 167º, Undershoot



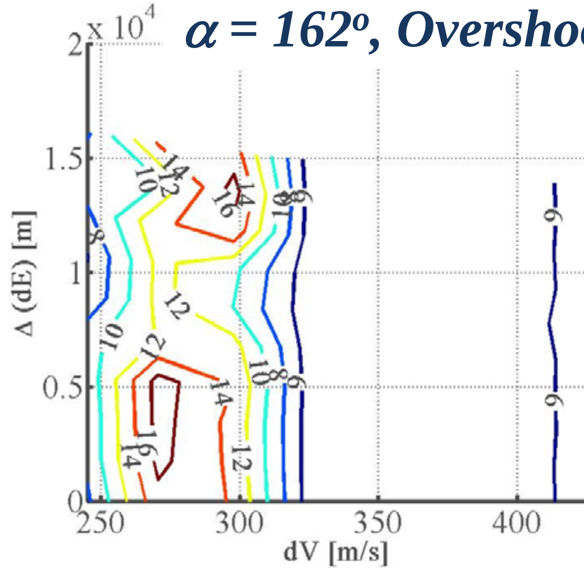
Targeting Algorithm Development

Design Space Interrogation, Results: *Bank Rate [deg/s]*

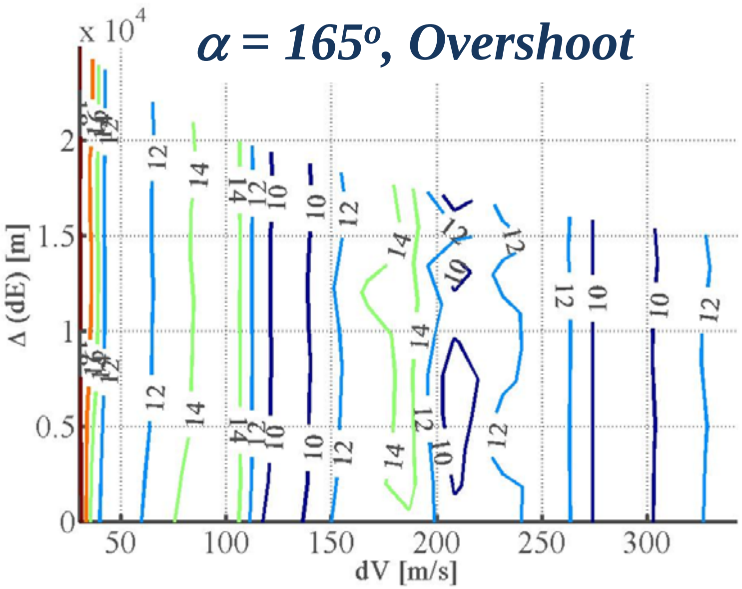
α = 152°, Undershoot



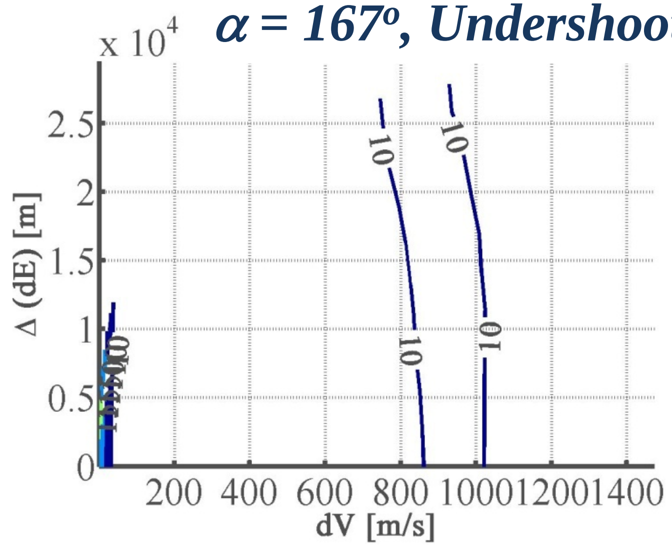
α = 162°, Overshoot



α = 165°, Overshoot



α = 167°, Undershoot



Targeting Algorithm Development Results

Dispersions –

Apollo Derived Guidance = -20% dispersion

MDAO Algorithm = -43% dispersion

Managing heatload may be a challenge for dispersions greater than 20%

Conclusions

Guidance Specific (In-Flight)

✓ Determine flight feasible control vectors (control rate/acceleration constraints)

o Be highly robust to dispersions and perturbations

✓ Include a minimal number of mission dependent guidance parameters $\left.\frac{D}{\rho}\right|_{ref}$ *and* γ_{ref}

Vehicle Design Specific

- Be applicable to multiple
  - ○/✓ mission scenarios
  - ✓ vehicle dispersions

o Manage the entry heat load in addition to achieving a precision landing

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Dissertation Chair

Dr. Dean Karnopp
Dissertation Committee Member

UC Davis Mechanical and Aerospace Engineering Faculty

UC Davis Mechanical and Aerospace Engineering Staff

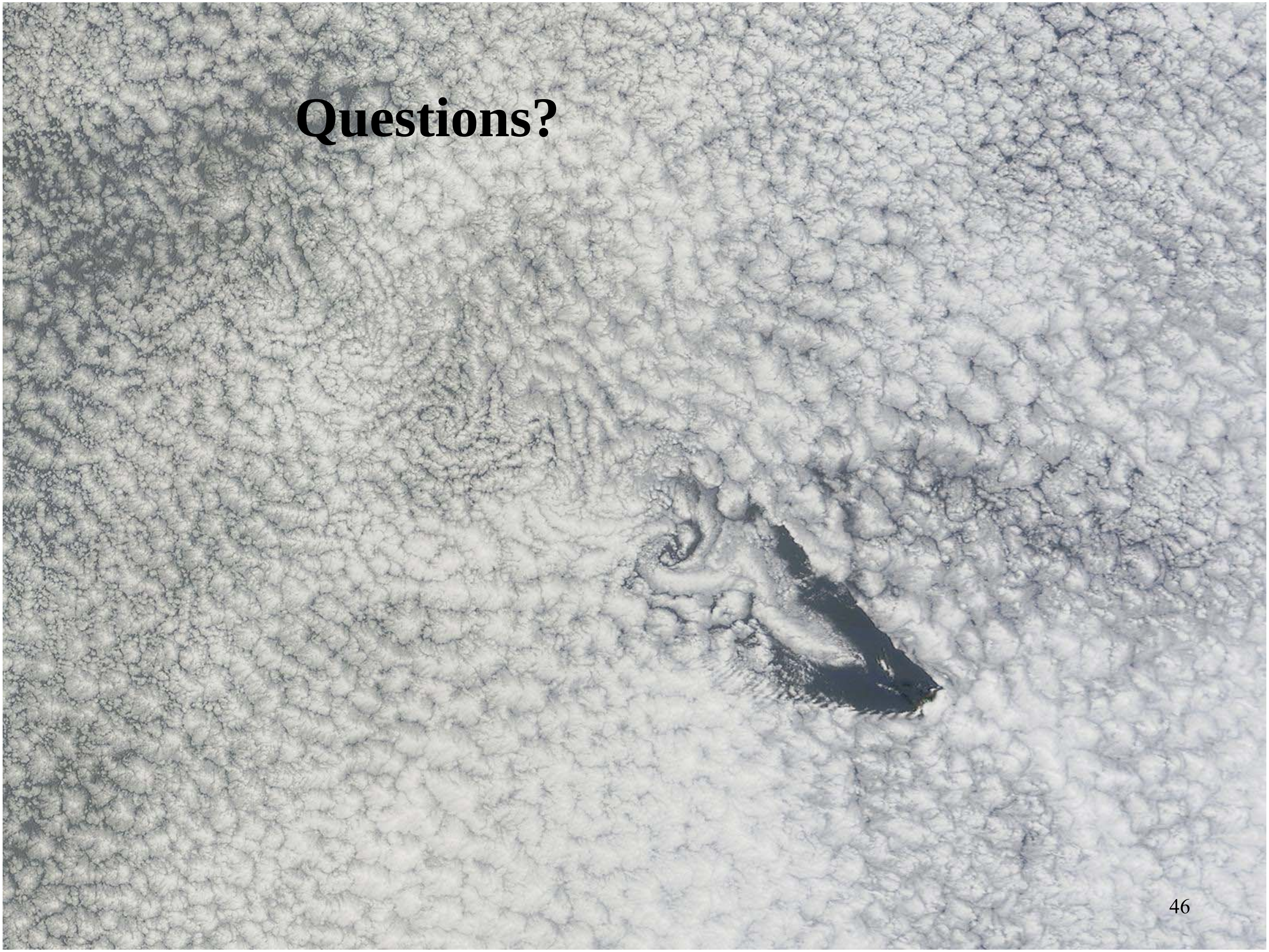
NASA Ames Research Center

Dr. Dave Kinney
Dissertation Committee Member

Mary Livingston
Supervisor

Colleagues in Systems Analysis Office

Thank You !!!



Questions?

Additional Slides
(optional)

Overview

Background & Motivation	Elements of Spacecraft Design
	Introduction to Planetary Entry Guidance
	Dissertation Research Plan and Status
MAPGUID Development	MAPGUID Proposed Approach
	Key Results #1
	Key Results #2
	Key Results #3
	Key Results #4
Aerothermal Management During Guidance	Proposed Approach
	Key Results #1
	Key Results #2
Guidance/COBRA Integration	Proposed Approach
	Key Results #1
	Key Results #2
	Key Results #3
Closing Remarks	Dissertation Findings and Status

Big Picture:
Spacecraft Design Process

MDAO Literature Review

Vehicle Optimization and TPS Sizing

Example Objective Function: $\dot{q}_{conv} = 1.83 * 10^{-4}\sqrt{\rho / R_n}(1 - h_W / H_s)V^3$

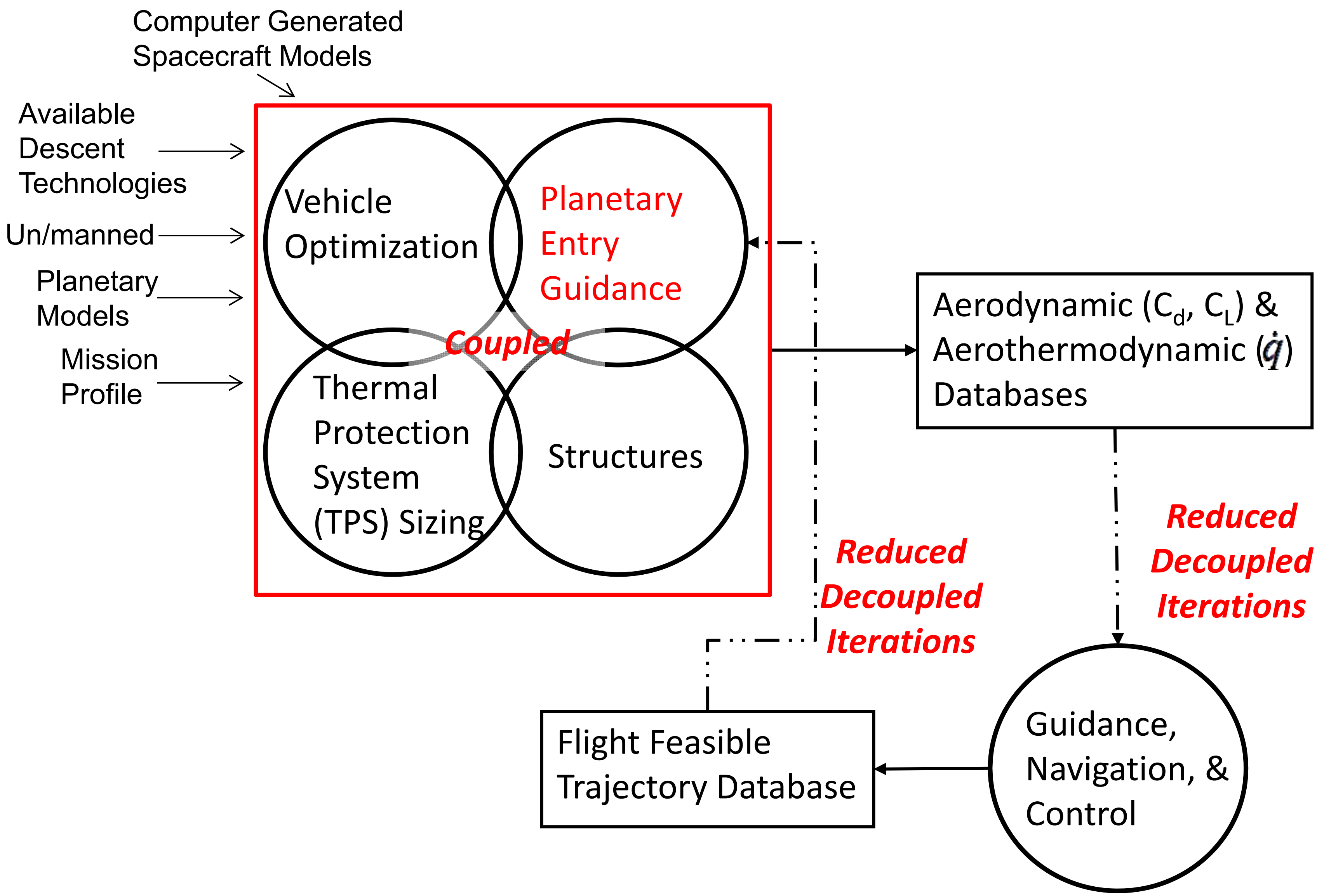
Results

- Most studi … corresponding to maximum …
  - Used for all geometries within optimization to find heat rate
- Some studies use new trajectories, but there is no accounting for bank constraints or target accuracy
- **None of these studies incorporated flight feasible trajectories**

What is Flight Feasible?

- Reaches Target @ Landing Speeds
- Control does not exceed system limits

Proposed Approach to MDAO for Spacecraft Design



Trajectory Modeling for Design vs. In-Flight Trajectory Modeling

Planetary Entry Guidance Literature Review

- **High L/D, Earth**: Space Shuttle, X-33, X40A
  - Most Robust: In Flight Trajectory Shaping with Reference Tracking
  - Least Robust: Reference Tracking Only
- **Low L/D, Earth**: Apollo, Orion
  - Most Robust: In-Flight Controls Search
  - Least Robust: Reference Tracking Only
- **Other Planetary Entry Vehicles**: MSR, MSL, Biconic
  - Flight Tested algorithms preferred

Planetary Entry Guidance Literature Review (cont’d)

Key Results

Modern guidance algorithms: optimal control is potential framework, but convergence still an issue

[illegible]

Trajectory Optimization Literature Review

Trajectory Optimization

Traj - Nonlinear constrained optimization

Mission - Sequential Quadratic Programming

Energy State Method – Reduced Order Modeling, one dimensional parameter search

Pseudospectral Methods – Combination indirect and direct method, mapping and discretization of domain

Trajectory Optimization Literature Review (cont’d)

Key Results

Curse of dimensionality: Convergence time increases with dimensionality

Fidelity of modeling may be compromised

Introduction to Planetary Entry Guidance

Guidance Development Process

Trajectory Design

Trajectory Optimization
-classical optimization
-genetic optimization

Bank Profile Scan
Determine constraint boundaries to identify flight corridors

Constant Bank

Modulated Bank
linear ramp, user-defined scans at each time step

Reference Trajectory Found

Guidance Development

Identify Target:
-Orbit
-Landing Site
-Descent Device Activation State
-Multiple

Trajectory Modeling in Real-Time to Predict Target Acquisition:
-Profiles Stored in Tables
-Analytical Equations
-Numerical Integration of EOMs
-Hybrid Approach

Targeting Algorithm:
-Parameterization
-One-Dimensional Bank Search
-Optimal Control Formulation
-Analytical Equation Blending
-Search Family of Trajectories
-Hybrid Approach

Apply Dispersions

Guided Trajectories Found

*Guidance must be robust to many dispersions:
(Atmospheric properties, Aerodynamics properties, Navigational Inputs, Entry Interface Conditions, Mass, Control System performance, and many others)

Baseline Vehicle & Mission

Case Study Parameters

Vehicle	Orion Capsule, L/D = 0.4
Trajectory	Skip Entry for Lunar Return
Control	Bank Angle only
Atmospheric Model	1976 Standard Atmosphere
Gravity Model	Central Force + Zonal Harmonics
Aerodynamics	C_L, C_D corresponding to Mach #
	CBAERO Databases, function of Mach #, Dynamic Pressure, and Angle of Attack
Trajectory Simulation	MATLAB Simulation validated against SORT Trajectories

Trajectory Simulations Developed

Open Loop Numerical Predictor- Corrector (NPC) Simulation

Used to test guidance formulations

3DOF Rotating Spherical Planet

$$\dot{r} = V\sin\gamma \qquad \dot{\theta} = \frac{V\cos\gamma\sin\psi}{r\cos\phi} \qquad \dot{\phi} = \frac{V\cos\gamma\cos\psi}{r}$$

$$\dot{V} = -D - g\sin\gamma + \Omega^2 r\cos\phi(\sin\gamma\cos\phi - \cos\gamma\sin\phi\cos\psi)$$

$$\dot{\gamma} = \frac{1}{V}\left[L\cos\sigma + \cos\gamma\left(\frac{V^2}{r} - g\right) + 2\Omega V\cos\phi\sin\psi + \Omega^2 r\cos\phi(\cos\gamma\cos\phi + \sin\gamma\cos\psi\sin\phi)\right]$$

$$\dot{\psi} = \frac{1}{V}\left[\frac{L\sin\sigma}{\cos\gamma} + \frac{V^2}{r}\cos\gamma\sin\psi\tan\phi - 2\Omega V(\tan\gamma\cos\psi\cos\phi - \sin\phi) + \frac{r\Omega^2}{\cos\gamma}\sin\psi\sin\phi\cos\phi\right]$$

Flight Simulation - Closed Loop Guidance Testing

Using equations derived from Newton’s 2nd Law, dynamics of relative motion, and Earth Centered Inertial (ECI) coordinate system

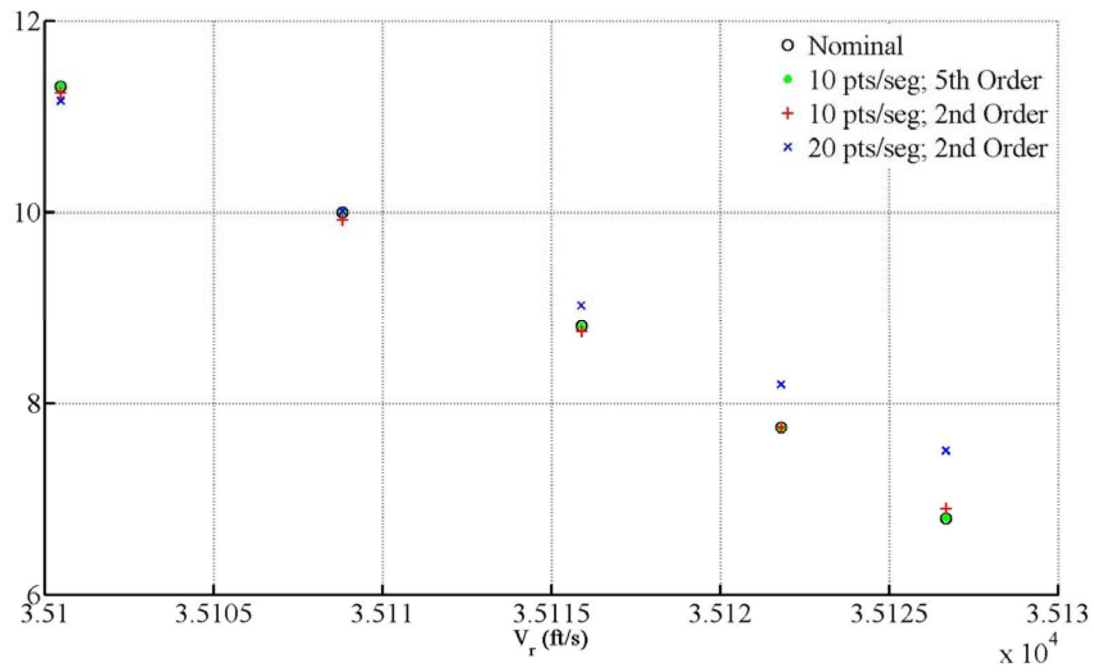
Trajectory Solver Development

Control Solution: Shuttle Entry Guidance Adaptation

Drag Curve Fit Accuracy

Segments	***Order***	***# of stored coefficients***
7 (3)	Irrational	168
7 (5)	Irrational	105
14	5	84
7	2	21

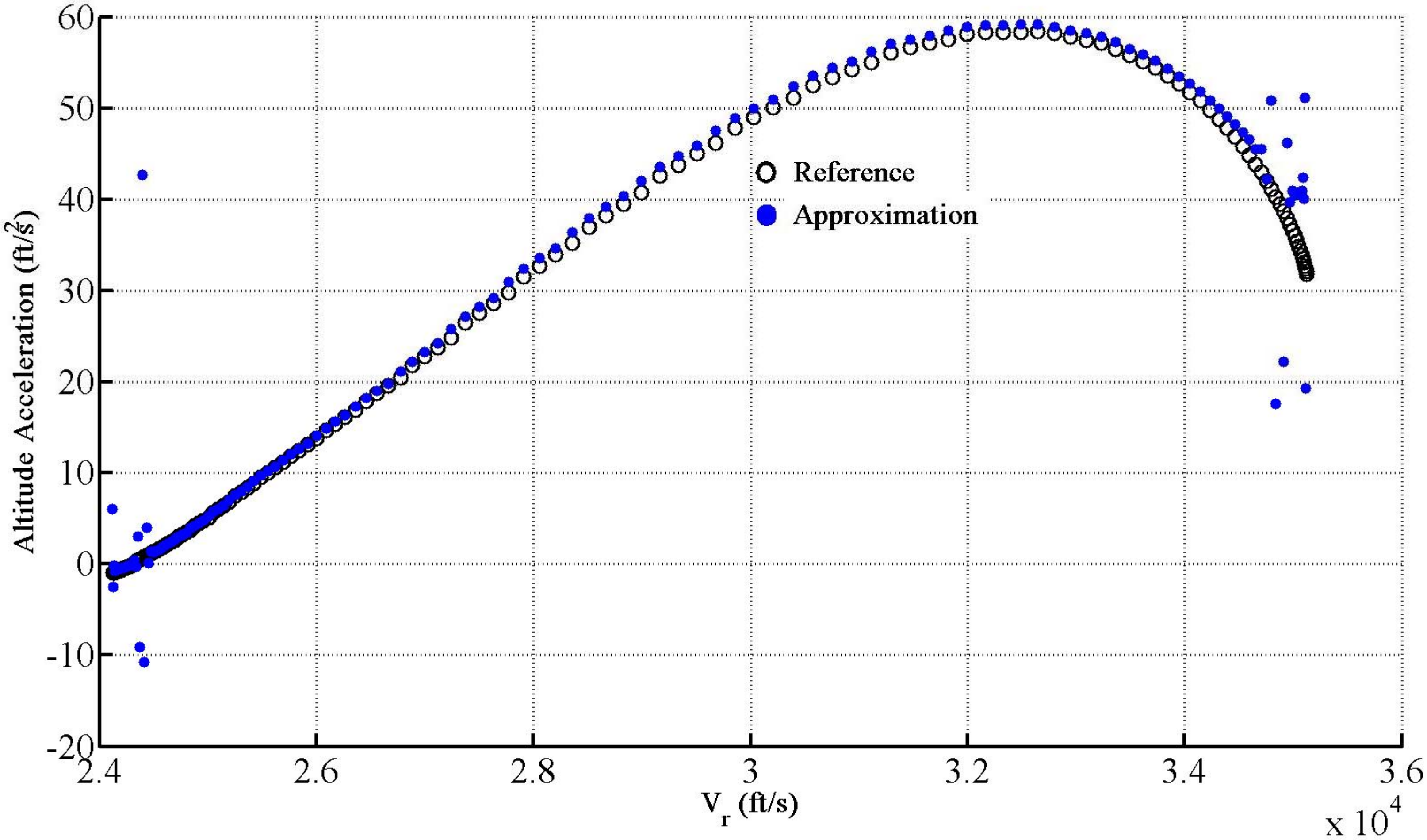
$$D_{ref} = C_2 V^{x_2} + C_1 V^{x_1} + C_0 V^{x_0}$$



Control Solution: Shuttle Entry Guidance Adaptation

Would Cubic Spline Interpolation work?

$$h_s = \left(\frac{1}{P}\frac{dP}{dh} - \frac{1}{T}\frac{dT}{dh} \right)^{-1}$$

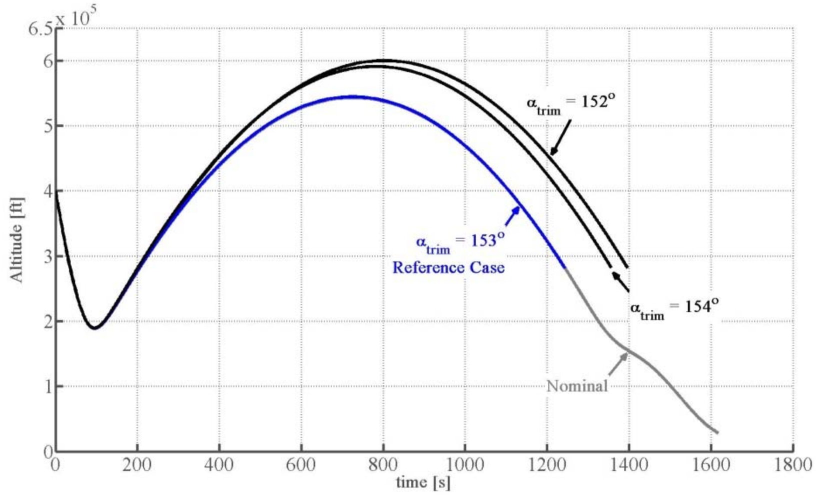


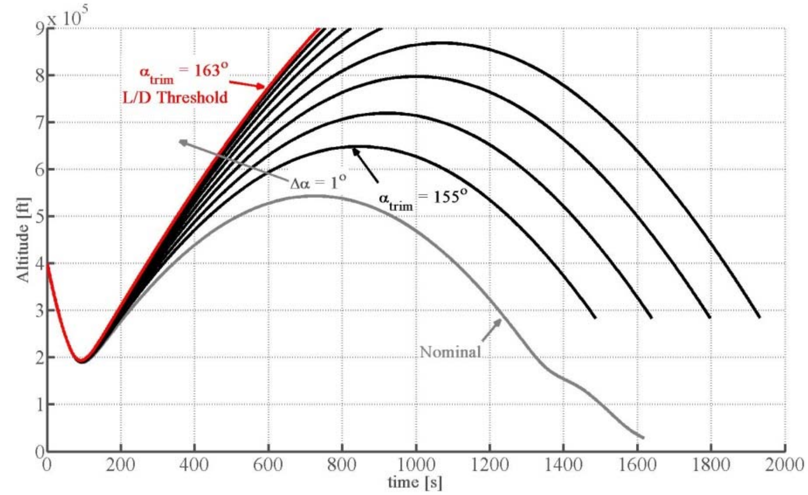
Targeting Algorithm Development

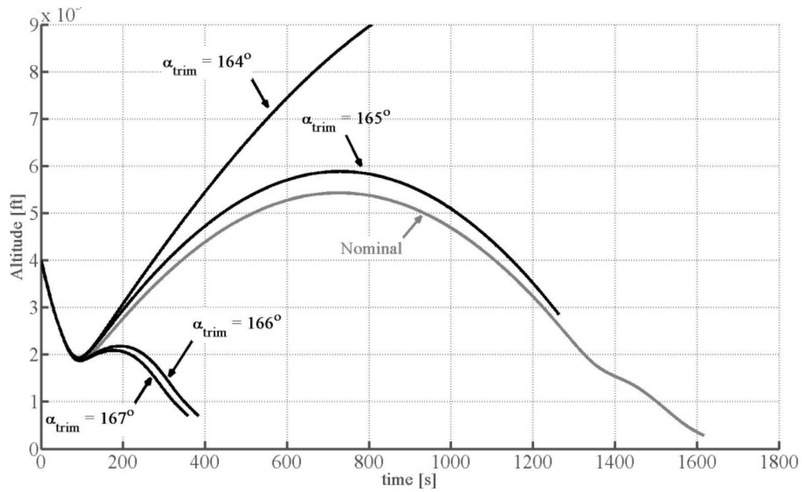
Targeting Algorithm Development

Targeting Technique 1 – Trajectory Behavior to Full Set of Aerodynamic Dispersion

Can Technique 1 find a trajectory that points toward correcting the range error?







General Conclusions

Targeting Algorithm Development

Pontryagin's Principle in Optimal Control

Find Optimal Control $\vec{u}^*$ $\sigma^*(t)$ and $V^*(t)$

for dynamic system $\dot{\mathrm{x}} = f(\vec{x}, \vec{u}, t)$

The optimal control $\dot{e} = \frac{V}{g_o}\dot{V} + \frac{h_{geo}^2}{h^2}\dot{h}$ raints including the Euler-Lagrange Equation:

$$\left.\frac{\partial H}{\partial \boldsymbol{u}}\right|_{\boldsymbol{u}=\boldsymbol{u}^*} = \mathbf{0}$$

$$\dot{\phi} = \frac{V\cos\gamma}{r} \quad \bar{\lambda} = \frac{\lambda_\psi}{\lambda_\gamma} = \tan\sigma^*\cos\gamma$$

$$\lambda\gamma \leq 0$$

$$\dot{\gamma} = \frac{1}{V}\left[L\cos\sigma + \cos\gamma\left(\frac{V^2}{r} - g\right) + 2\Omega V\cos\phi\sin\psi\right.$$
$$\left.+\Omega^2 r\cos\phi(\cos\gamma\cos\phi + \sin\gamma\cos\psi\sin\phi)\right]$$

$$\dot{\psi} = \frac{1}{V}\left[\frac{L\sin\sigma}{\cos\gamma} + \frac{V^2}{r}\cos\gamma\sin\psi\tan\phi - \right.$$
$$2\Omega V(\tan\gamma\cos\psi\cos\phi - \sin\phi) +$$
$$\left.\frac{r\Omega^2}{\cos\gamma}\sin\psi\sin\phi\cos\phi\right]$$

Targeting Algorithm Development

Targeting Technique 1

$$\bar{\lambda}_{new} = \bar{\lambda}_{old} \pm d\lambda$$

Determines new bank angle at current time step

σ new

$$\dot{\gamma} = \frac{1}{V}\left[L\cos\sigma + \cos\gamma\left(\frac{V^2}{r} - g\right) + 2\Omega V\cos\phi\sin\psi + \Omega^2 r\cos\phi(\cos\gamma\cos\phi + \sin\gamma\cos\psi\sin\phi)\right]$$

$$\Phi_{new,bound} = \frac{\sqrt{1+\left(\frac{\bar{\lambda}}{\cos\gamma}\right)^2}\left[V\dot{\gamma}_{ref} - \cos\gamma\left(\frac{V^2}{r} - g\right) - 2\Omega V\cos\phi\sin\psi - \Omega^2 r\cos\phi(\cos\gamma\cos\phi + \sin\gamma\cos\psi\sin\phi)\right]}{\rho\frac{L}{D}_{total}}$$

$$\Phi_{new} = \Phi_{old} \pm C_{\Phi}|\Phi_{old} - \Phi_{new,bound}|$$

Calibrated for Each Dispersed Case

$$V_{initial} = V_{current} + 0.01 dV$$

$$V_{ref,f} = V_{current} - (1 - 0.01)dV$$

Determines Blended Trajectory that nulls range error

Targeting Algorithm Development

Targeting Technique 1 – Design Space Interrogation

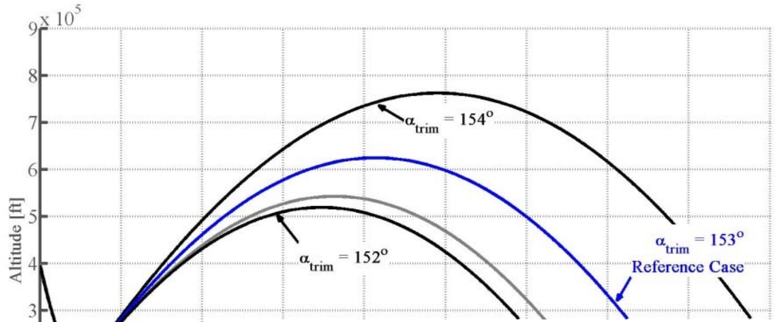
- The blending technique exhibits potential to find new bank profiles that null the range error

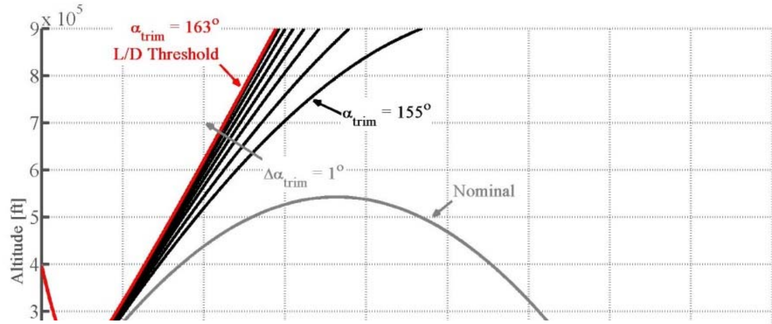
- The design space is constrained by control system limitations

- There is a zero range error solution for each change in $d\lambda$

Targeting Algorithm Development

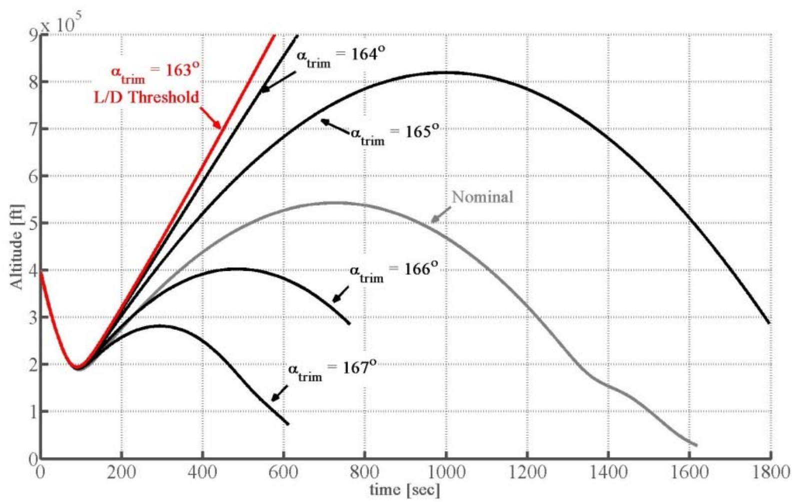
Targeting Technique 1 – Trajectory Behavior to Full Set of Aerodynamic Dispersion





Why did this not follow the Expected Behavior?

The reference bank profile over-corrects with respect to the dispersion of L/D



Targeting Algorithm Development

Targeting Technique 2

Now that the blended function is fully defined $\Phi_{blnd} = Bb_2V^2 + Bb_1V + Bb_0$

The following equation can be used to solve for:

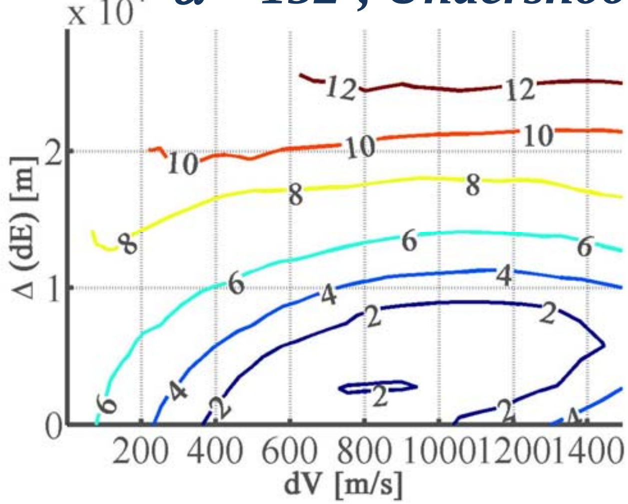
$$\dot{\gamma}_{i,new} = \frac{1}{V_i}\left[\frac{L}{D}\Big|_{total,i} \sqrt{\frac{1}{1+\left(\frac{\bar{\lambda}}{\cos\gamma_i}\right)^2}} \rho_i \Phi_{blnd,i} + 2\Omega V_i \cos\phi_i \sin\psi_i + \right.$$
$$\left. \cos\gamma_i \left(\frac{V_i^2}{r_i} - g_i\right) + \Omega^2 r_i \cos\phi_i (\cos\gamma_i \cos\phi_i + \sin\gamma_i \cos\psi_i \sin\phi_i)\right]$$

The FPA rate table is shifted accordingly

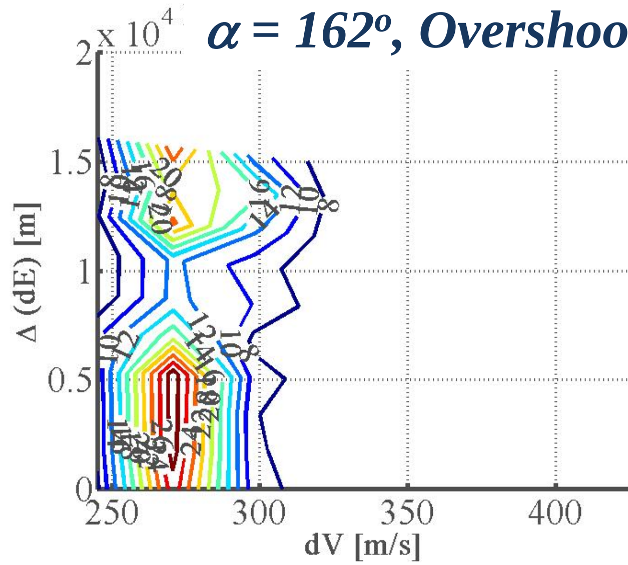
Targeting Algorithm Development

Design Space Interrogation, Results: *Bank Acceleration [deg/s^2]*

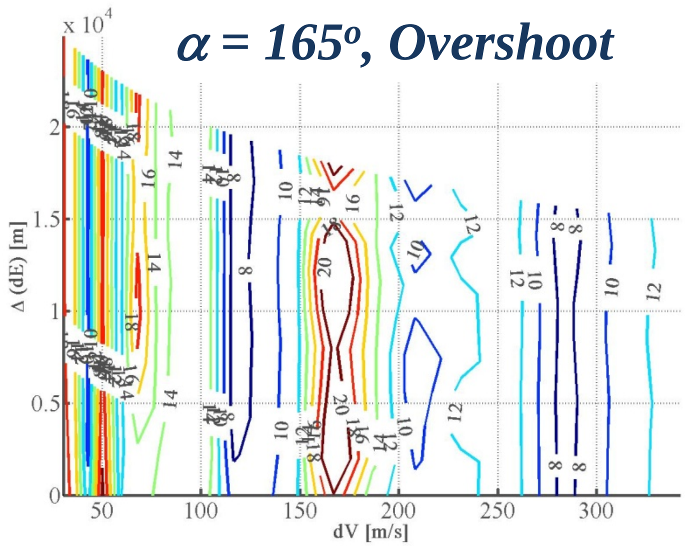
α = 152^o, Undershoot



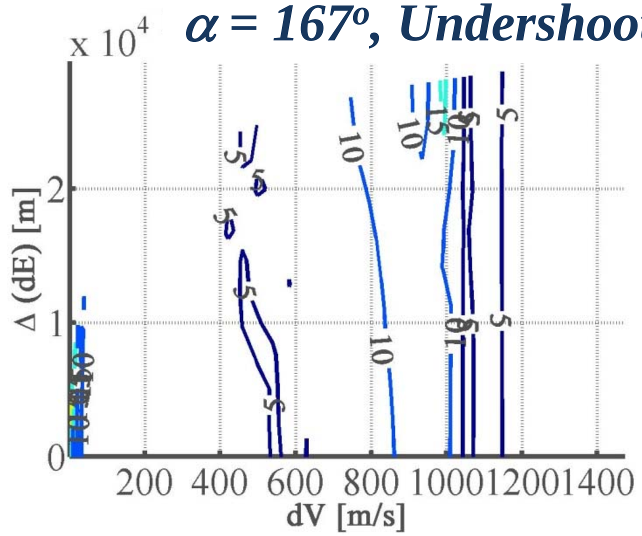
α = 162^o, Overshoot



α = 165^o, Overshoot



α = 167^o, Undershoot



Targeting Algorithm Development

Targeting Technique 1 – Targeting Implementation, 1st and 2nd Phase

1. Guess a value for $d\lambda$
2. Iterate on dV using secant method to converge on a zero range error trajectory
3. If no solution is found $d\lambda$ is incremented and the iteration is repeated
4. Solution is then flown in flight simulation

Performance Metric –

Compare range of aerodynamic dispersions this algorithm can handle to the range of aerodynamic dispersions a heritage algorithm can handle.

Trajectory Solver Research Questions

Can a simplification in the equations of motion be made without loss of accuracy?

Can a simplification on flight path angle be made without loss of accuracy?

Simplified Equations of Motion Study

3DOF Rotating , Spherical Earth (3RSP)

$$\dot{\psi} = \frac{1}{V}\left[\frac{L\sin\sigma}{\cos\gamma} + \frac{V^2}{r}\cos\gamma\sin\psi\tan\phi \underbrace{- 2\Omega V(\tan\gamma\cos\psi\cos\phi - \sin\phi) + \frac{r\Omega^2}{\cos\gamma}\sin\psi\sin\phi\cos\phi}_{\text{Coriolis and Centripetal Acceleration}}\right]$$

3DOF Non-Rotating Spherical Planet

$$\dot{\psi} = \frac{1}{V}\left[\frac{L\sin\sigma}{\cos\gamma} + \frac{V^2}{r}\cos\gamma\sin\psi\tan\phi\right]$$

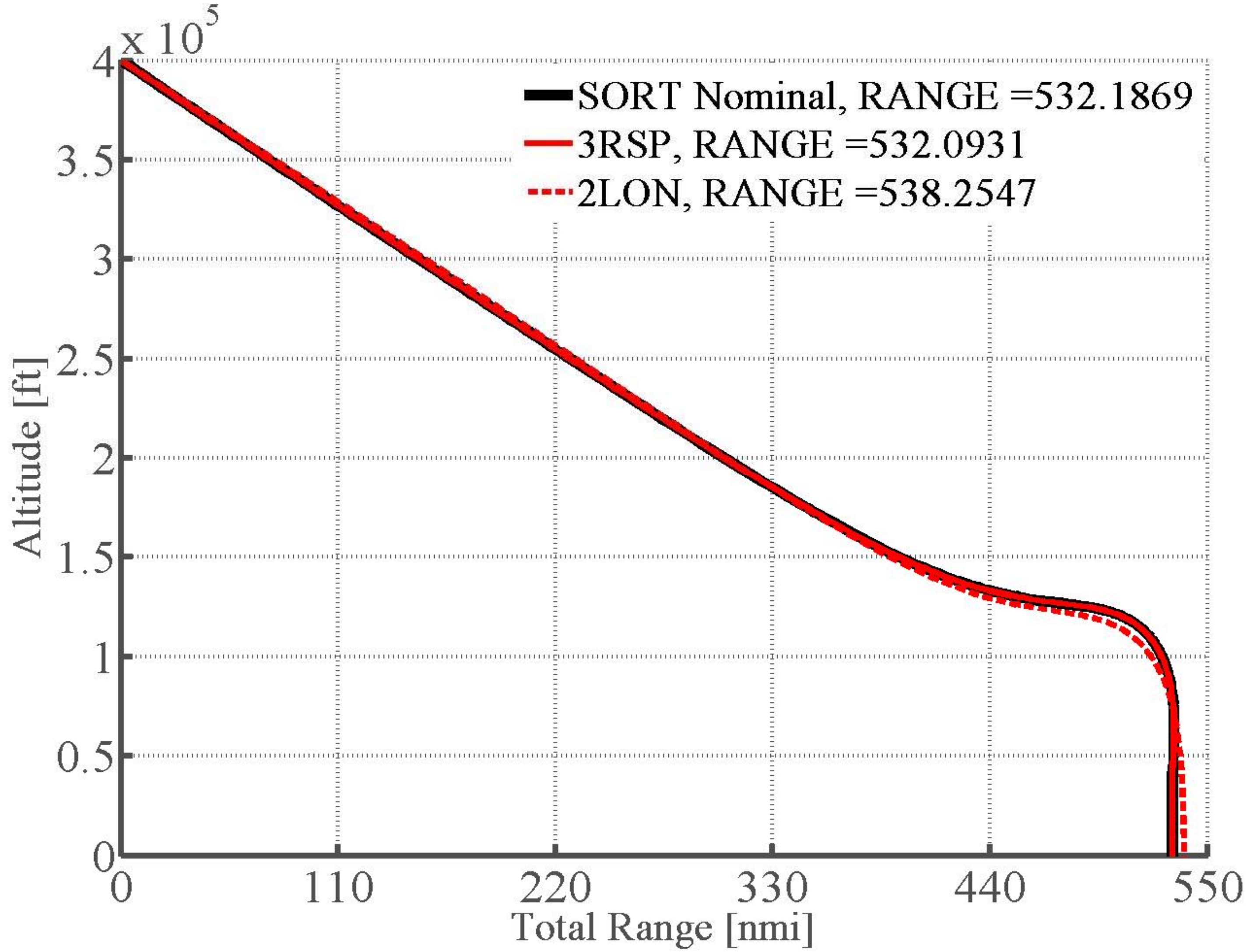
3DOF Non-Rotating Flat Planet

$$\dot{\psi} = \frac{1}{V}\left[\frac{L\sin\sigma}{\cos\gamma}\right]$$

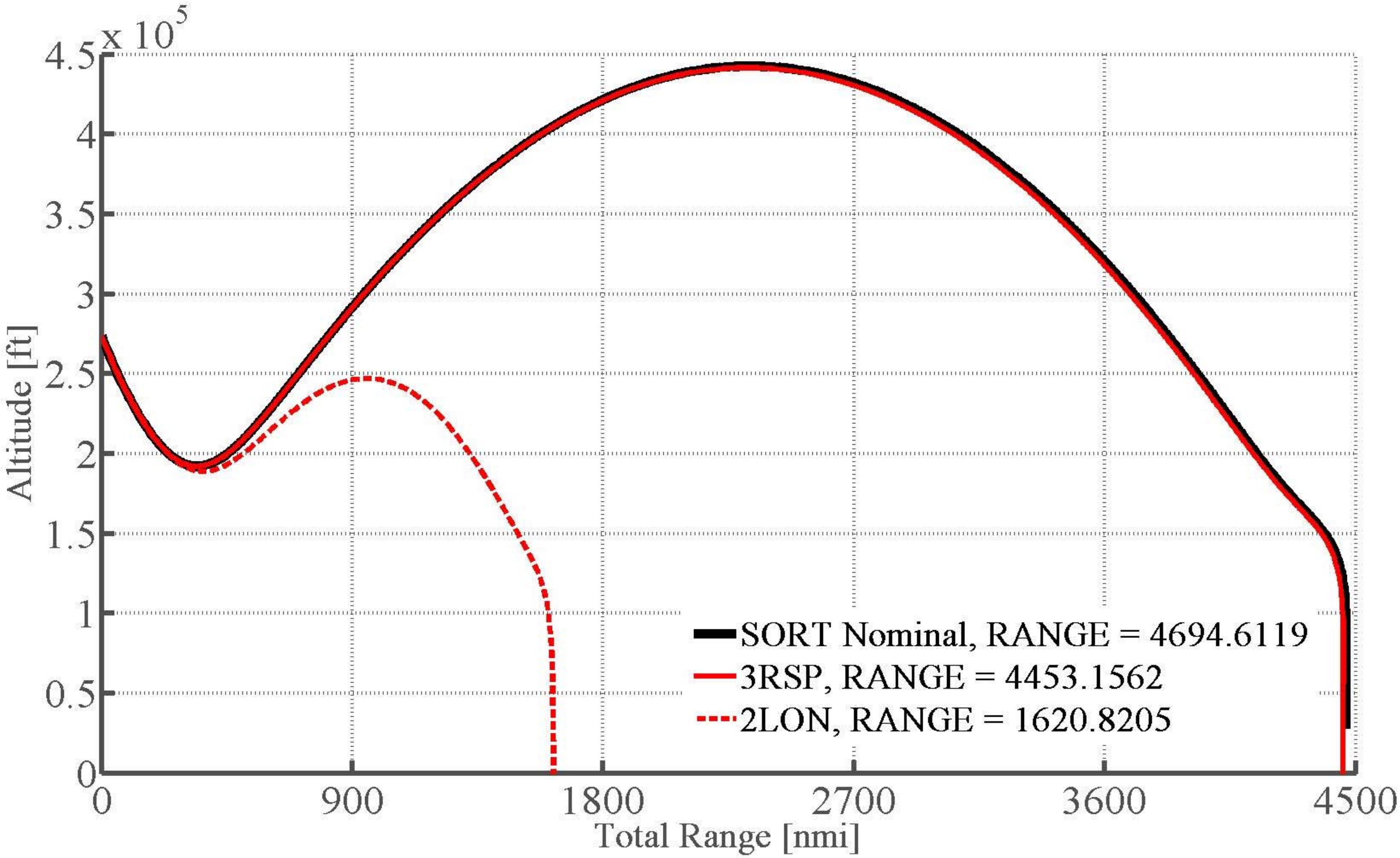
2DOF Longitudinal Equations (2LON)

$$\left.\begin{aligned}
\dot{h} &= V\sin\gamma \\
\dot{s} &= V\cos\gamma \\
\dot{V} &= -D - g\sin\gamma \\
\dot{\gamma} &= \frac{1}{V}\left[L\cos\sigma + \cos\gamma\left(\frac{V^2}{r} - g\right)\right]
\end{aligned}\right\} \text{Apollo and Shuttle Entry Guidance}$$

Simplified Equations of Motion Study (cont' d)



Simplified Equations of Motion Study (cont’d)



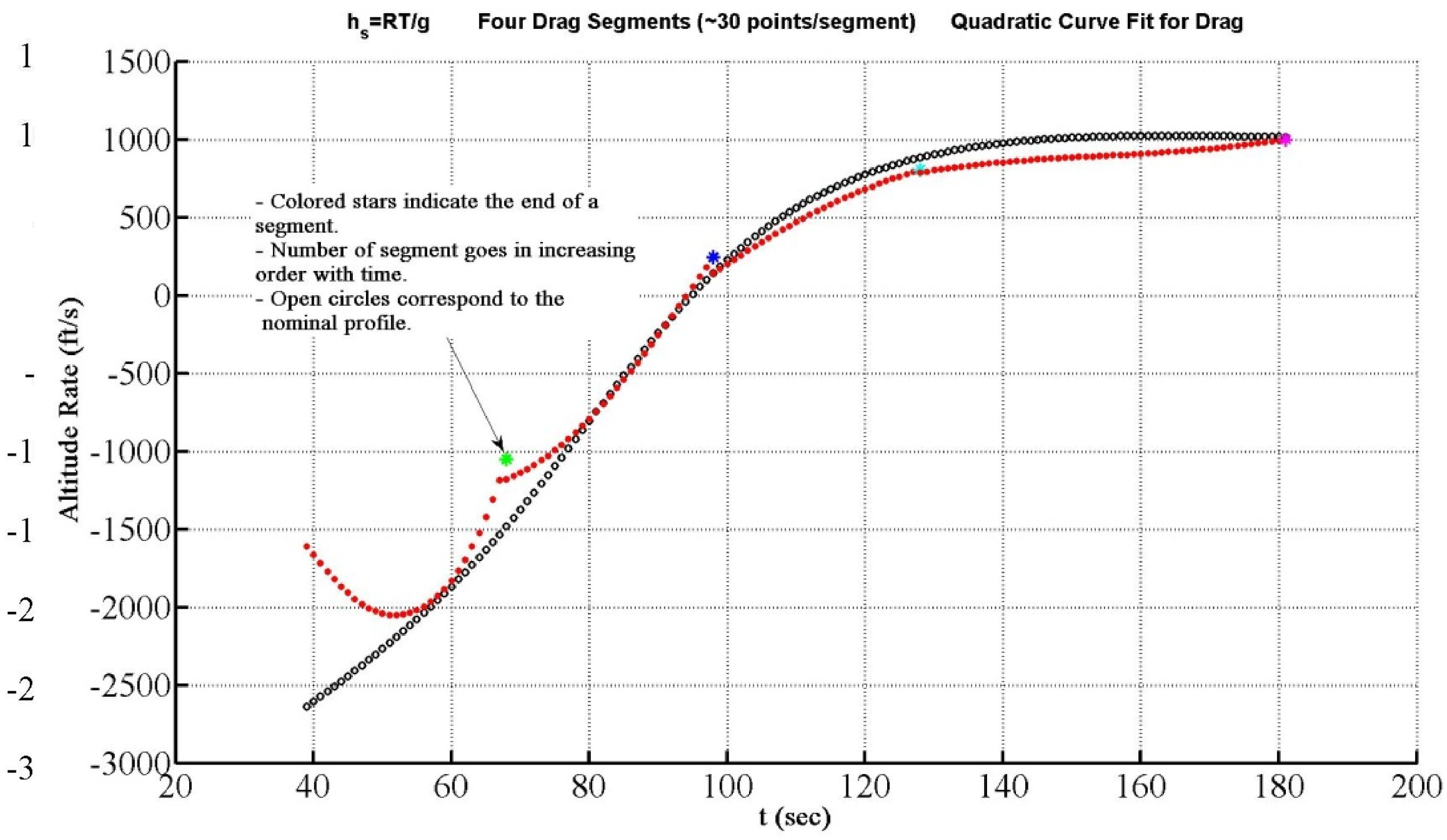
Trajectory Solver Research Questions

Can a simplification in the EOMs be made without loss of accuracy?
Not for a skip trajectory

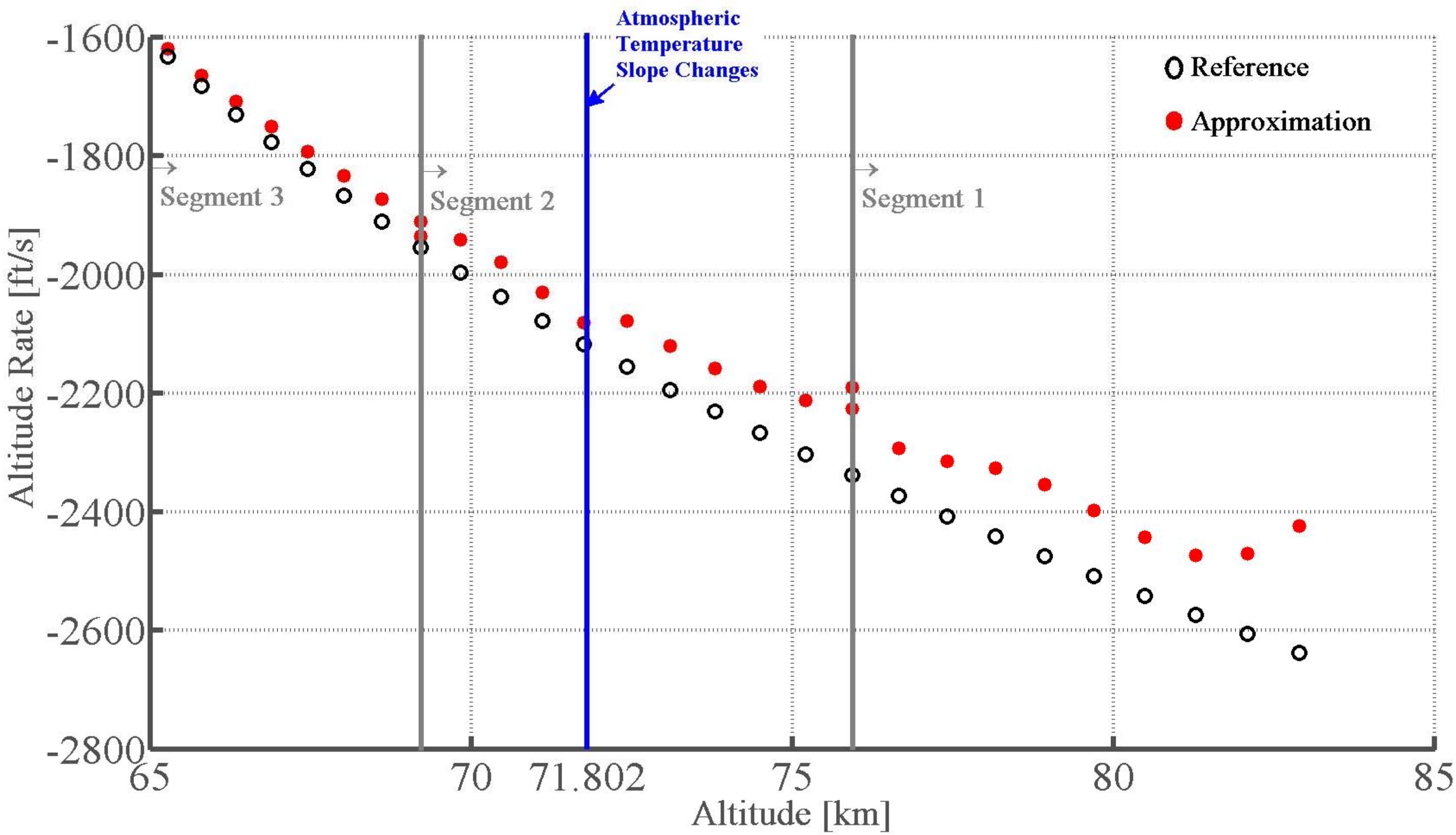
Can a simplification on flight path angle be made without loss of accuracy?

Control Solution: Shuttle Entry Guidance Adaptation

$$\dot{h}_{ref} = -h_s \left[\frac{\dot{D}_{ref}}{D_{ref}} - \frac{2\dot{V}}{V} - \frac{\dot{C}_D}{C_D} \right]$$



Control Solution: Shuttle Entry Guidance Adaptation

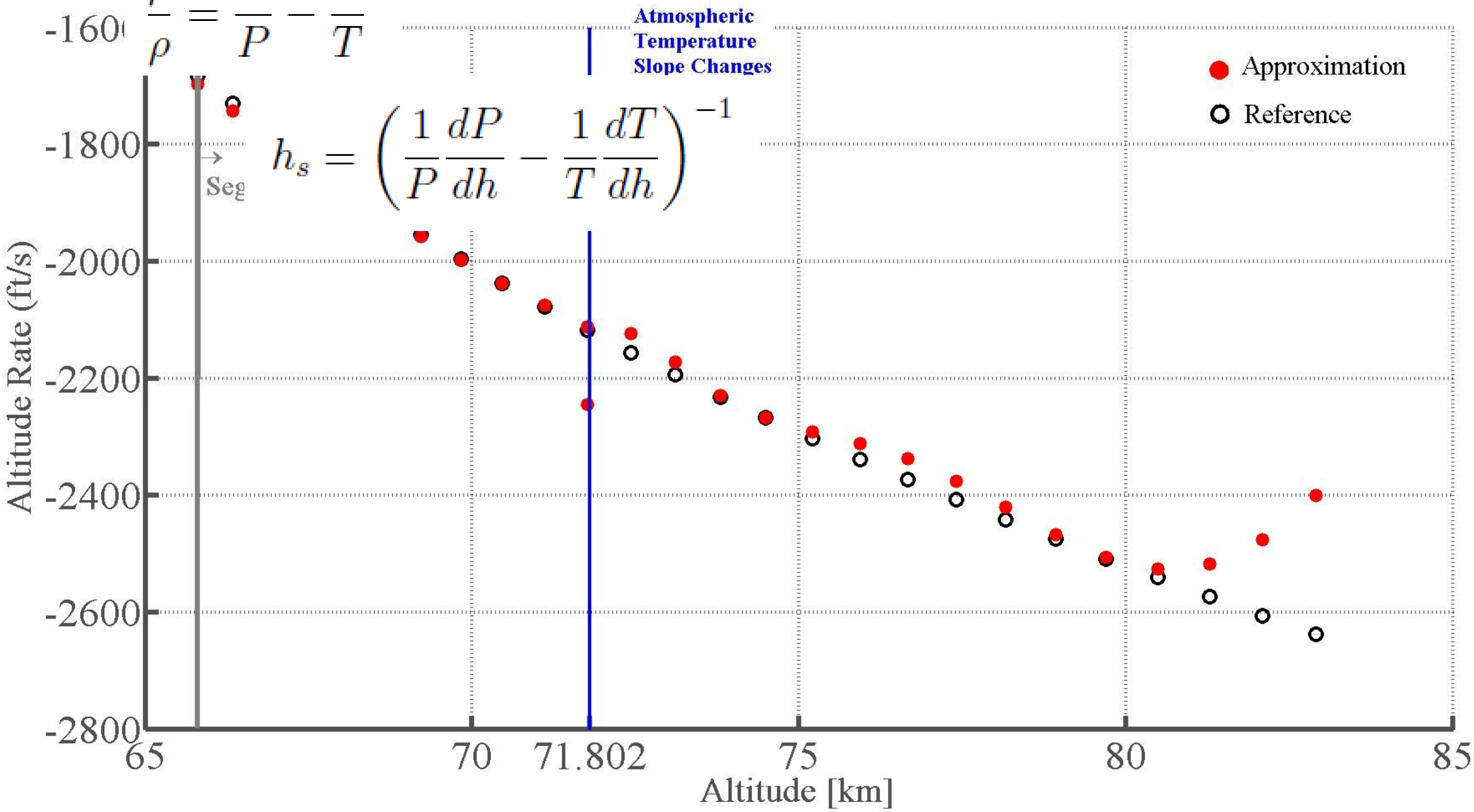


Control Solution: Shuttle Entry Guidance Adaptation

$$P = \rho R T$$

$$\frac{\dot{\rho}}{\rho} = \frac{\dot{P}}{P} - \frac{\dot{T}}{T}$$

$$h_s = \left(\frac{1}{P}\frac{dP}{dh} - \frac{1}{T}\frac{dT}{dh} \right)^{-1}$$

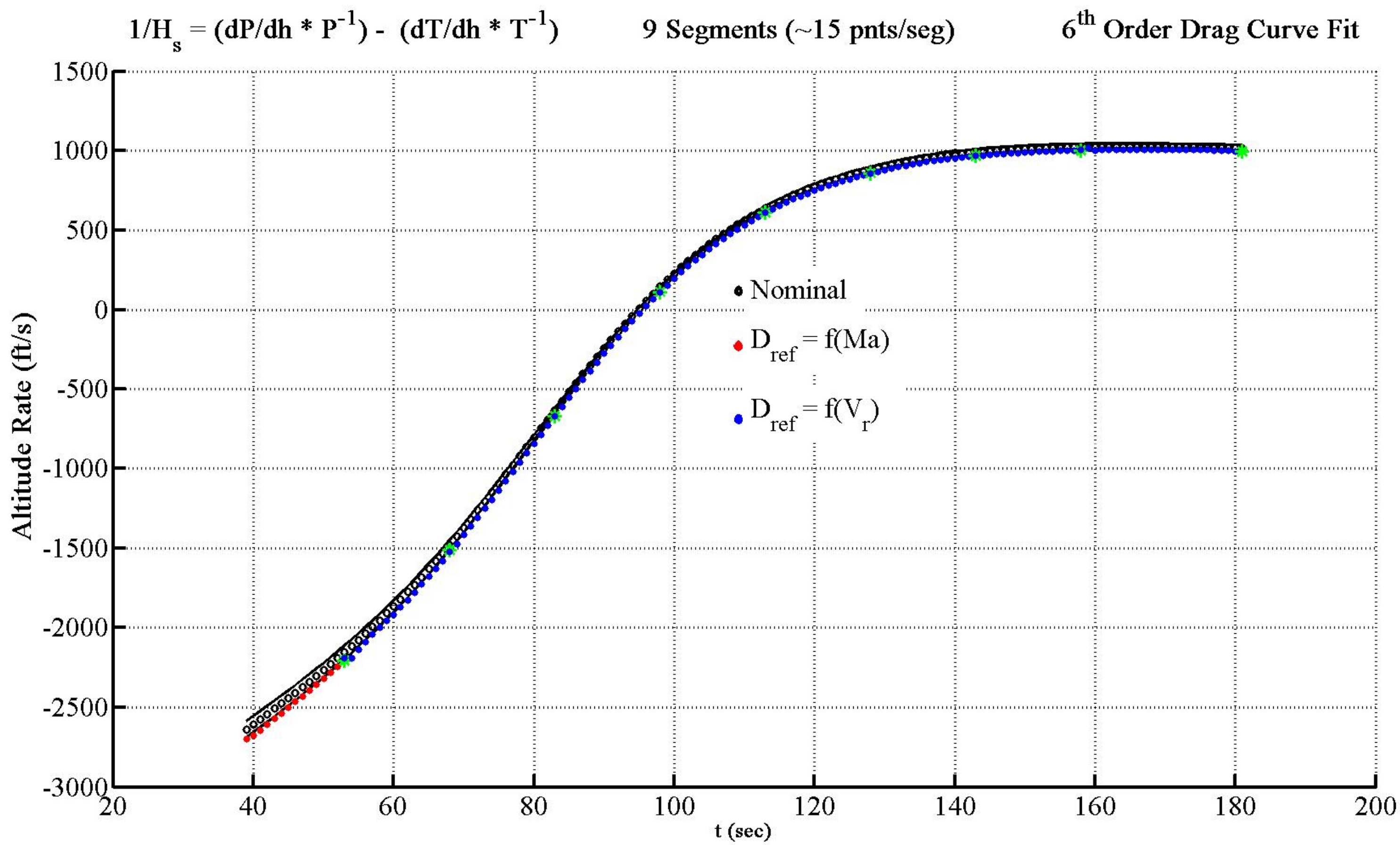


Control Solution: Shuttle Entry Guidance Adaptation

Need to Resolve 1st Segment to Capture Atmospheric Non-Linearity

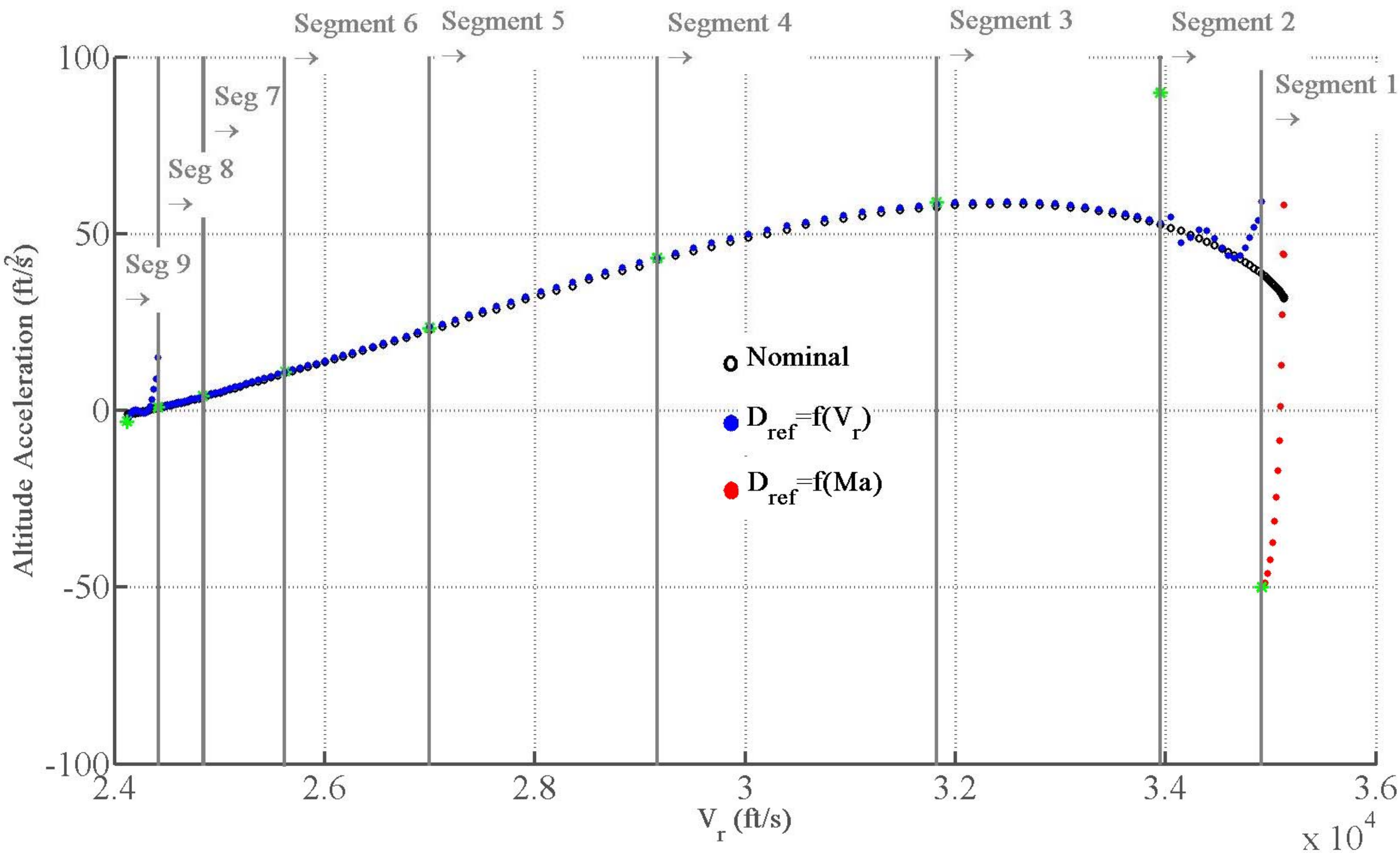
IDEA: Curve fit drag with Mach Number

$$D_{ref} = \sum_{i=1}^{n} C_i Ma^i$$



Control Solution: Shuttle Entry Guidance Adaptation

Check Altitude Acceleration Approximation



Trajectory Solver Research Questions

Can a simplification in the EOMs be made without loss of accuracy?
Not for a skip trajectory

Can a simplification on flight path angle be made without loss of accuracy?

Range Prediction Sensitivity to Flight Path Angle Assumption

- Apollo and Shuttle Entry guidance formulations approximate flight path angle (FPA) to be small: $\longrightarrow$ $\gamma << 1\, rad$ and/or $\dot{\gamma} << 1\, rad/s$

Why does this matter?

- If predicted range does not equal the range to landing site then targeting is erroneously active
- Are model reductions in the Trajectory Module and Control Module valid based on the nominal case?

Range Prediction Sensitivity to Flight Path Angle Assumption

Case Studies:

A. Apply $\gamma << 1\, rad$ to Trajectory Module only

Trajectory Module
NPC Solves 3DOF EOMs

B. Apply $\gamma << 1\, rad$ to Controls Module only

Controls Module
Drag and FPA Rate Reference Trajectories

C. Apply $\dot{\gamma} << 1\, rad/s$ to bank equation only

$$\left.\frac{L}{D}\right|_{v,ref} = \frac{1}{\rho\Phi_{ref}}\left[V_r\dot{\gamma}_{ref} - \cos\gamma\left(\frac{V_r^2}{r} - g\right) - C_\gamma(\mathrm{y})\right]$$

Range Prediction Sensitivity to Flight Path Angle Assumption

Nominal 661.73 [nmi]

Case	Total Range [nmi]	% Range Error	Termination
A	662.39	**0.099%**	*Drag Limit*
B	649.74	1.813%	*Drag Limit*
C	632.13	4.474%	*Velocity Limit*

Conclusion *FPA* approximation can be applied to the **trajectory module**, but not to the **control module**